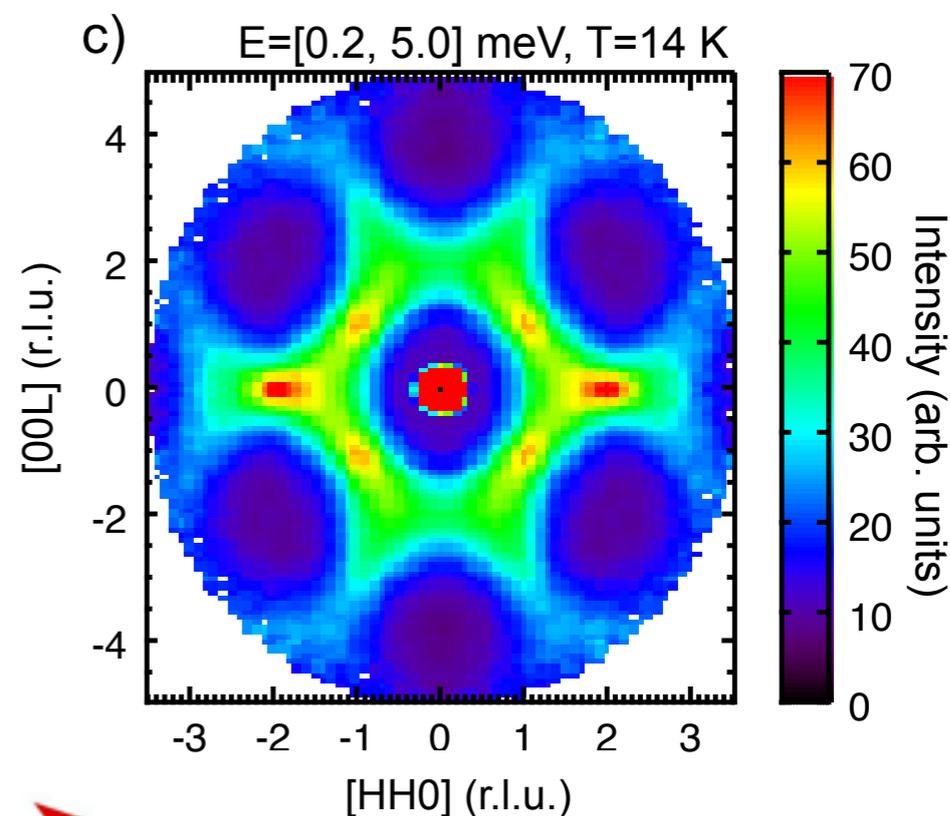
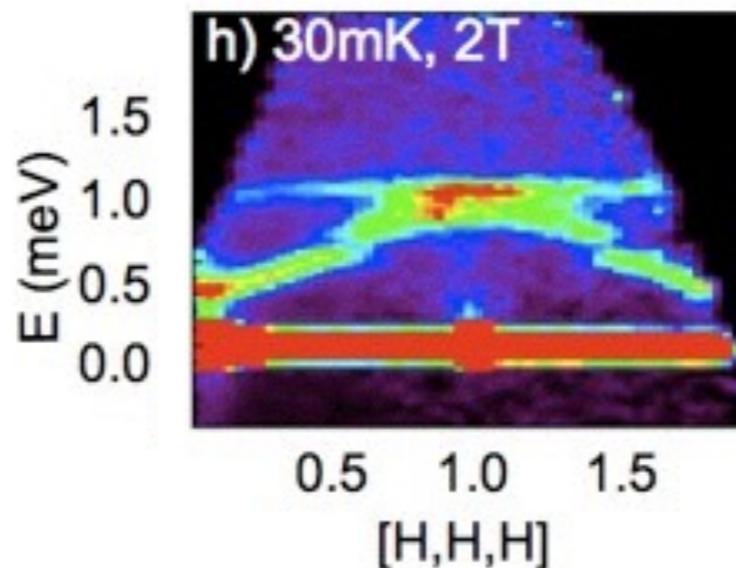
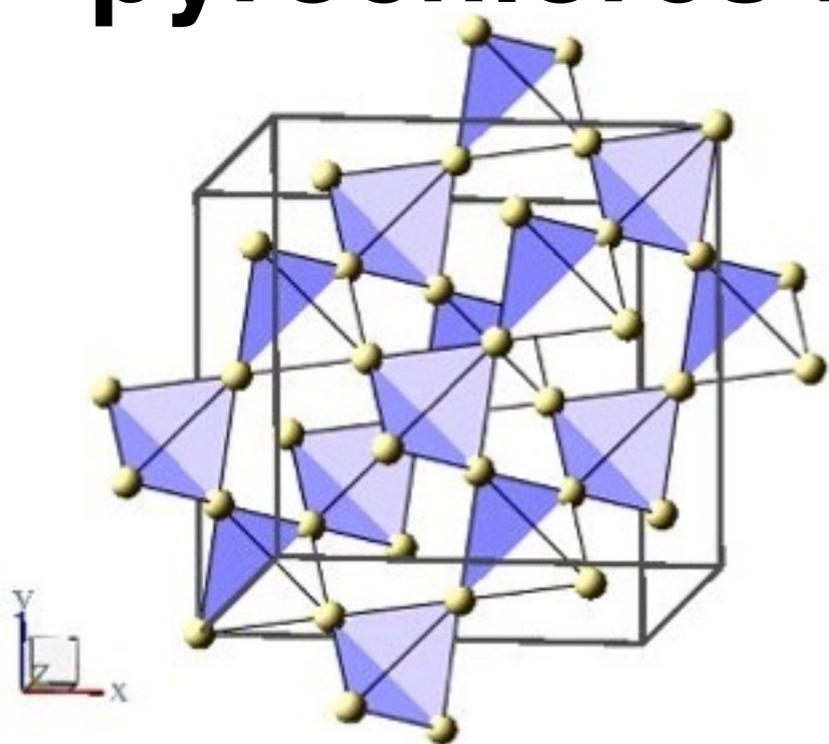


Quantum phenomena in $S_{\text{eff}} = 1/2$ pyrochlores revealed by neutron scattering



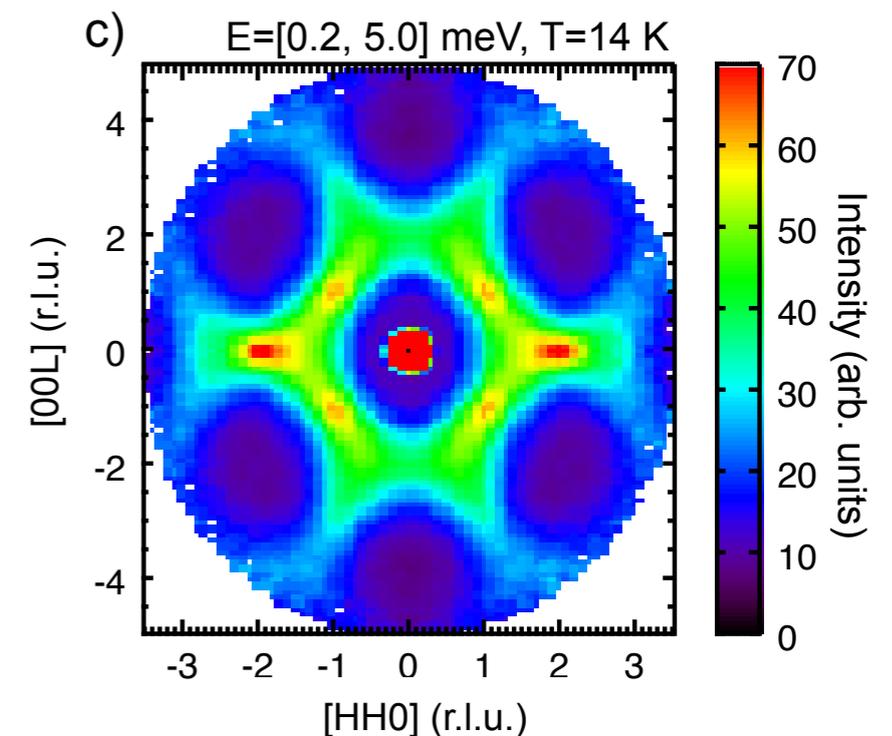
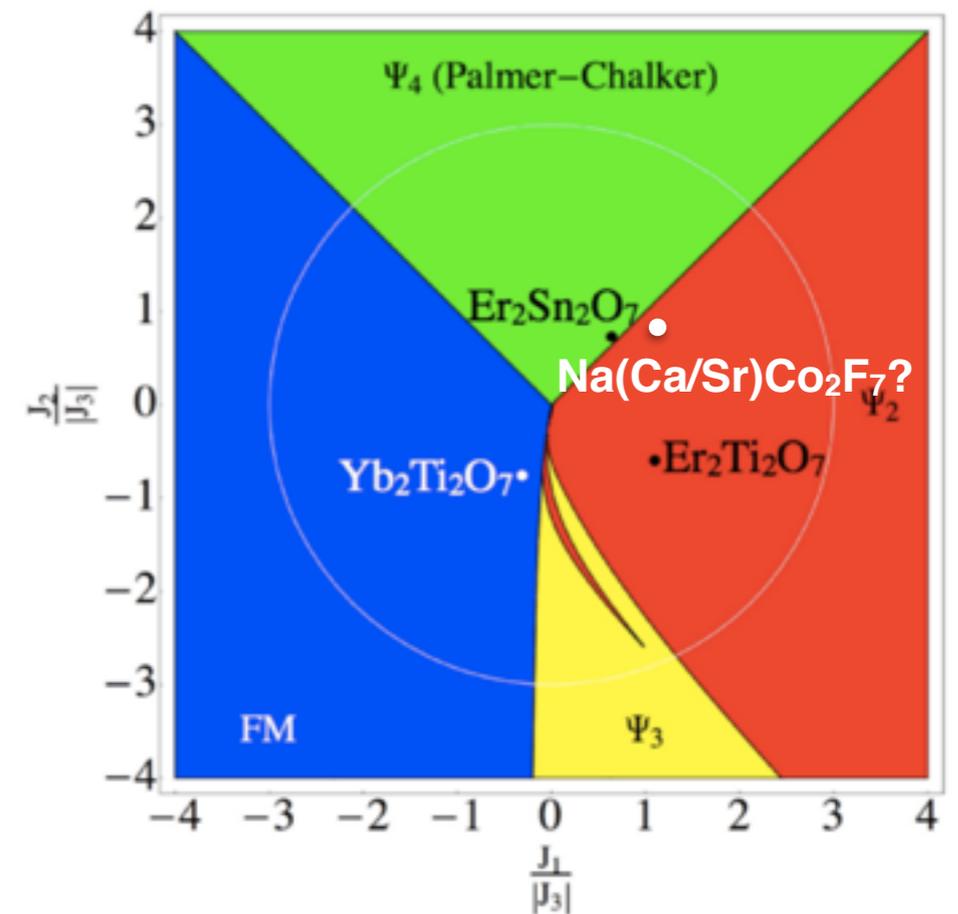
Kate A. Ross

Colorado State University

NCNR Neutron Scattering Summer School
NIST, June 22, 2017

Outline

- Geometric Frustration
- General anisotropic exchange phase diagram for pyrochlores
- Phase competition in real pyrochlores: **quantitative understanding using neutrons**
- $\text{Yb}_2\text{Ti}_2\text{O}_7$, $\text{Er}_2\text{Ti}_2\text{O}_7$, and $\text{NaCaCo}_2\text{F}_7$



Overchoice

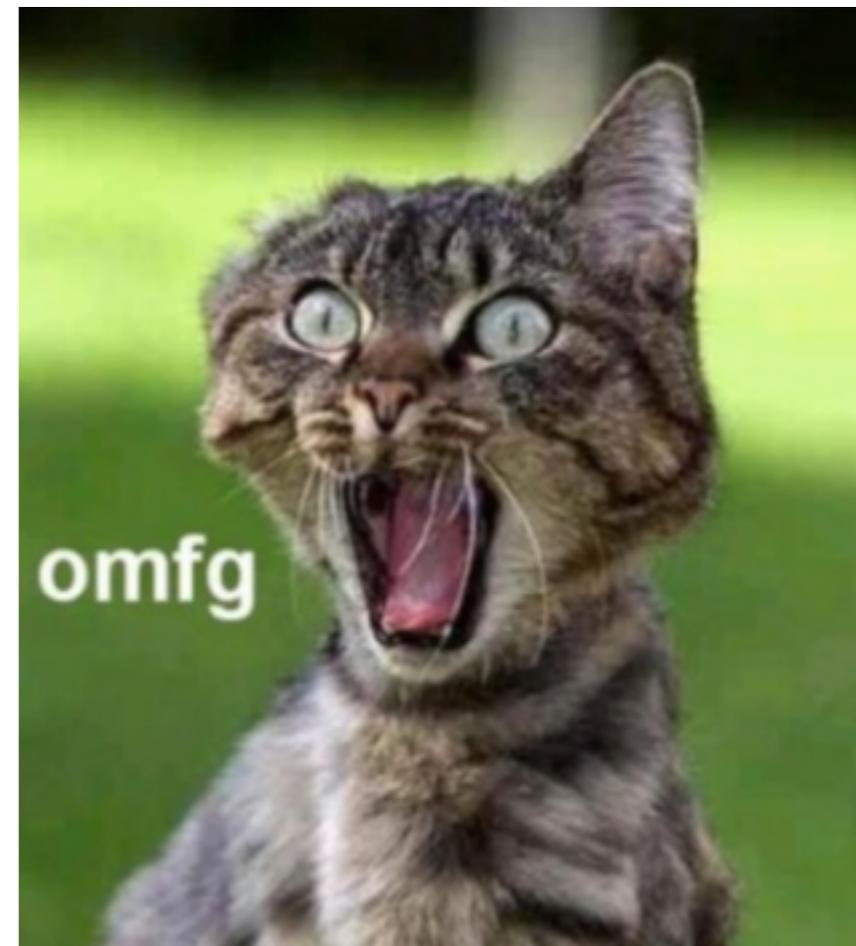
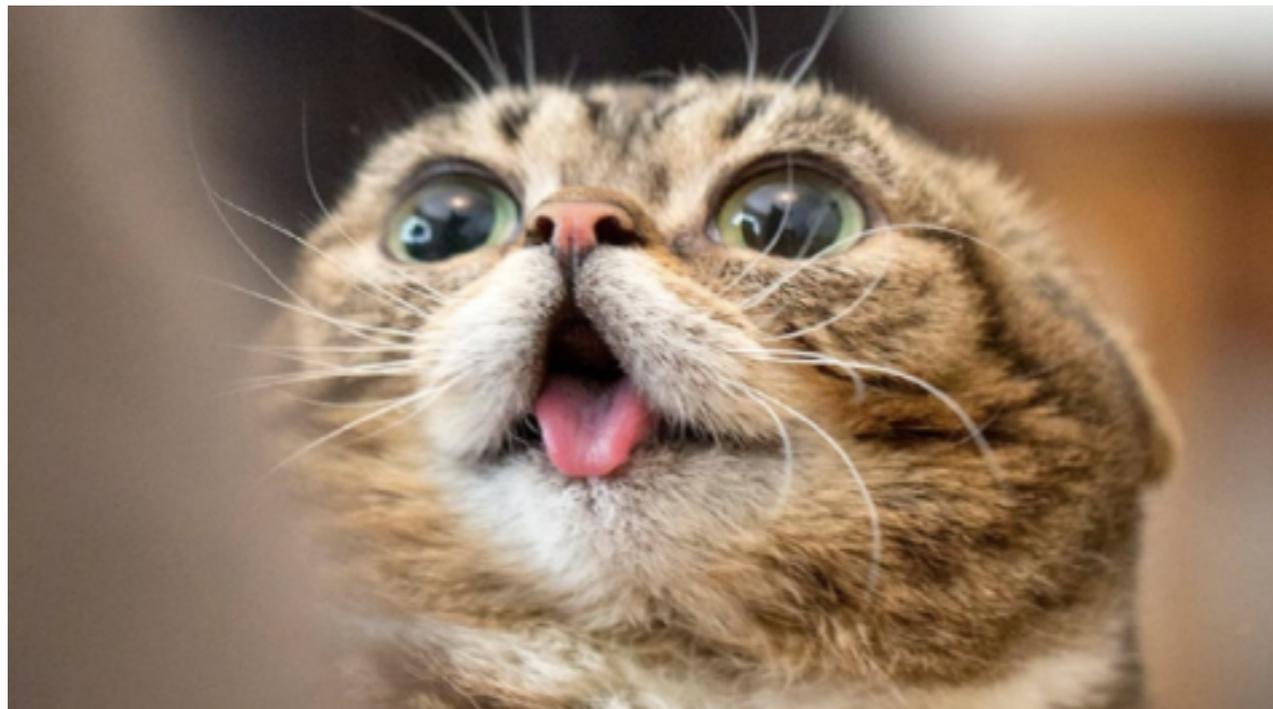


- Wikipedia: “**Overchoice** or **choice overload** is a cognitive process in which people have a difficult time making a decision when faced with many options.”

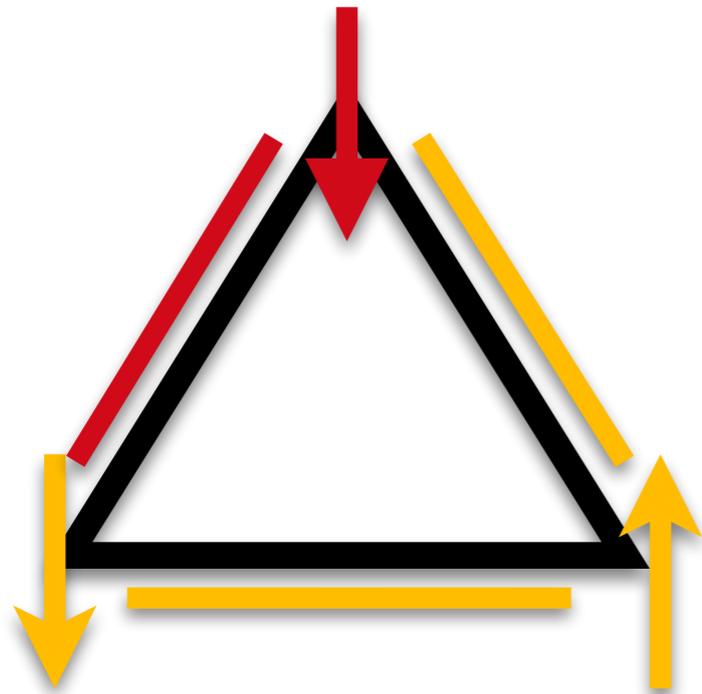
Overchoice = Frustration



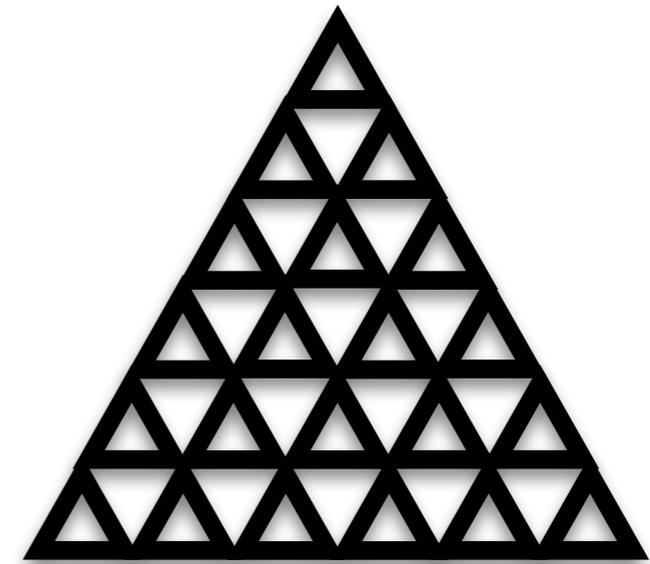
- **Frustration** from overchoice leads to interesting excited states



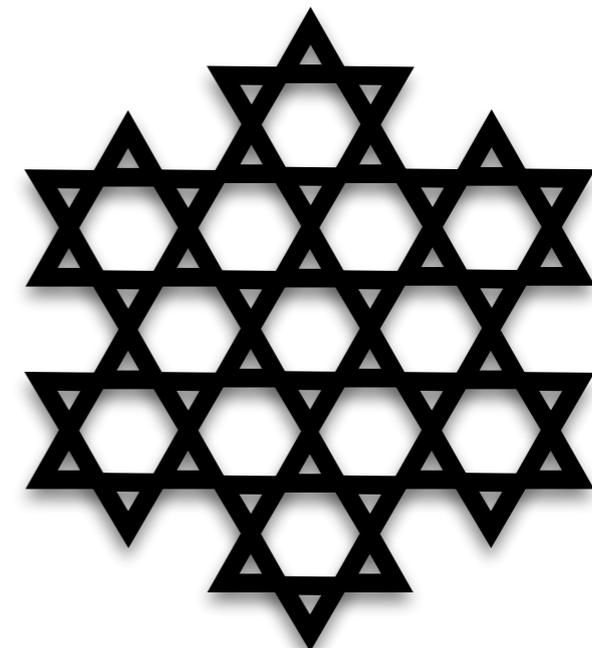
Geometric Frustration in 2D magnets



prefer $\uparrow \downarrow$ alignment,
but choice of 3rd spin
direction is unclear

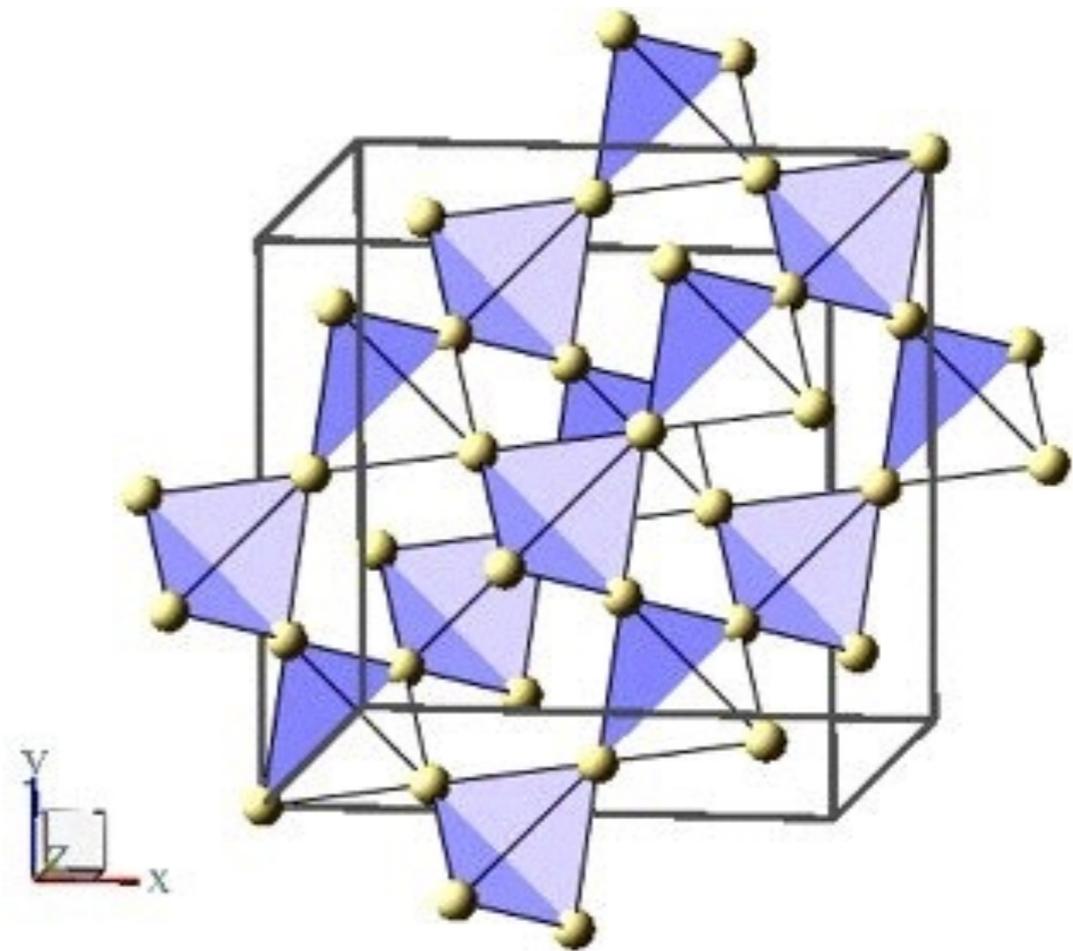


Triangular



Kagome

Geometric Frustration in 3D magnets



Pyrochlore

freedom of choice for each tetrahedron leads to a macroscopic degeneracy: **NO Long Range Order**

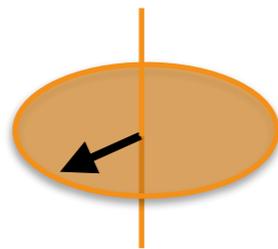
Local Anisotropy on the Pyrochlore Lattice

Types of Anisotropy

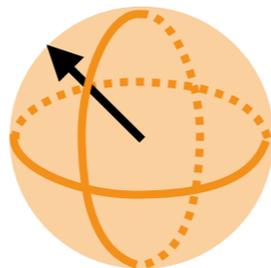
Ising



XY

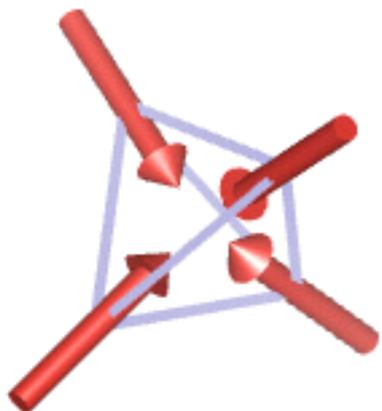


Heisenberg

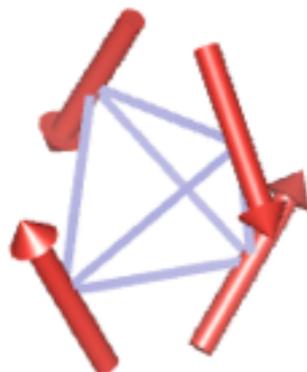


Some anisotropic AFM configurations

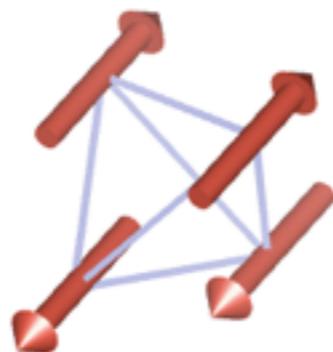
Ising



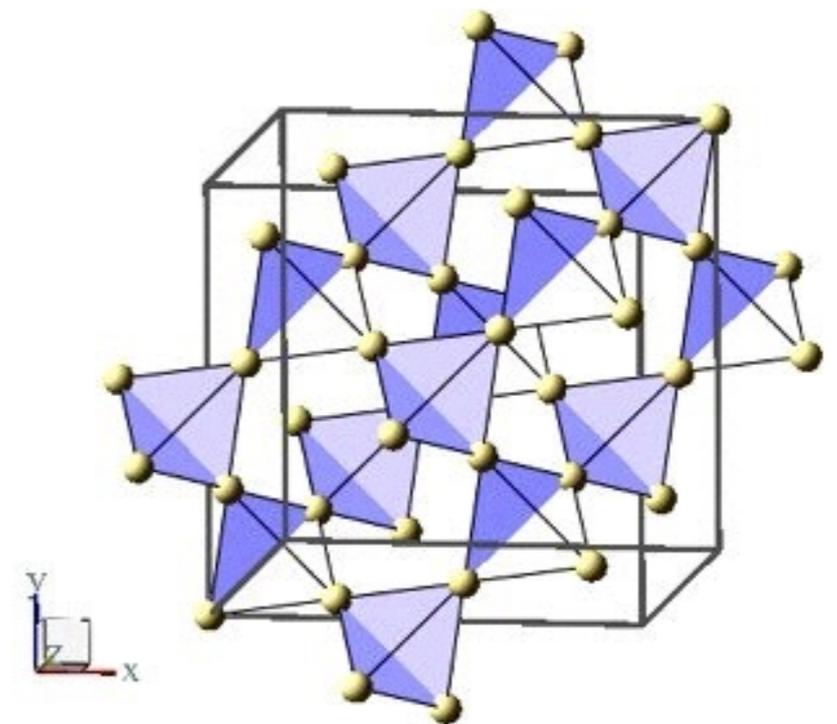
XY



Heisenberg



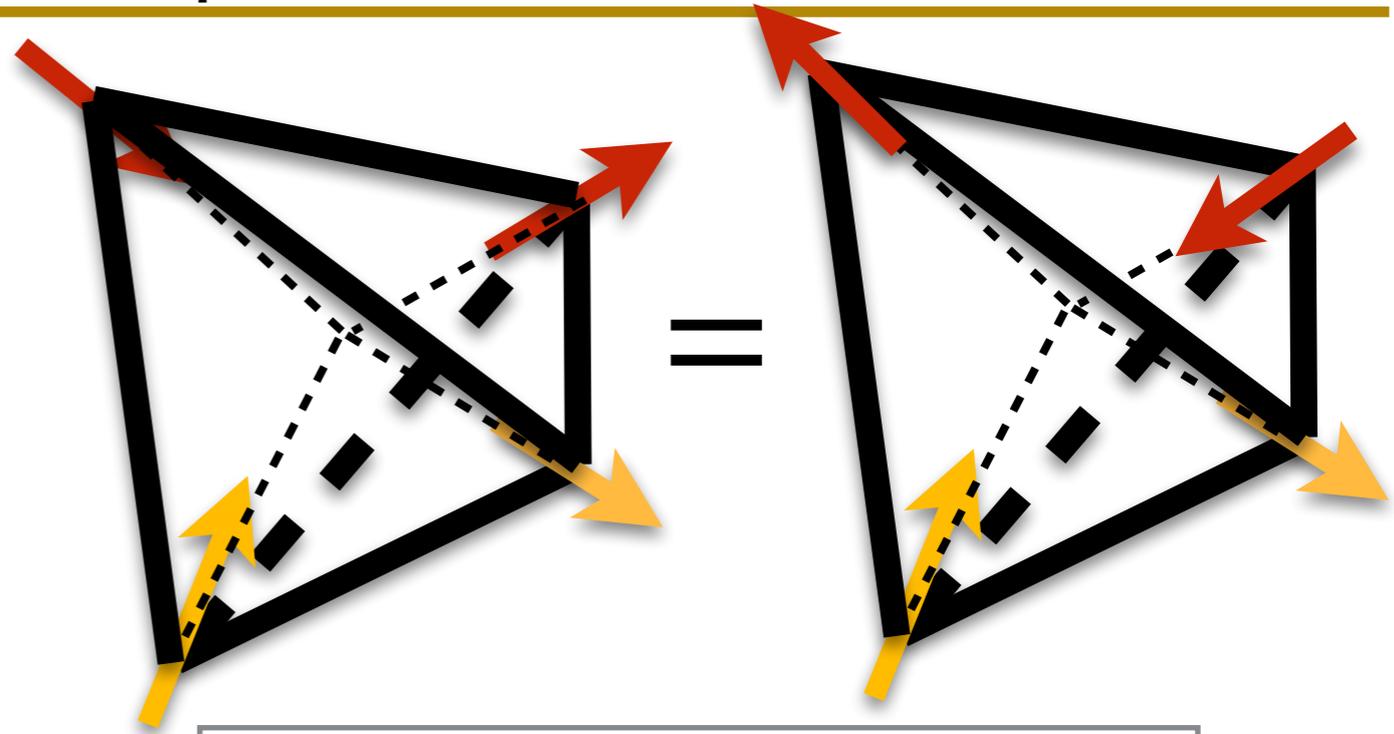
- crystal symmetry requires local axes for each sublattice
- “z” (Ising) is along *local* $\langle 111 \rangle$ (“In-to” or “out-of” tetrahedron)



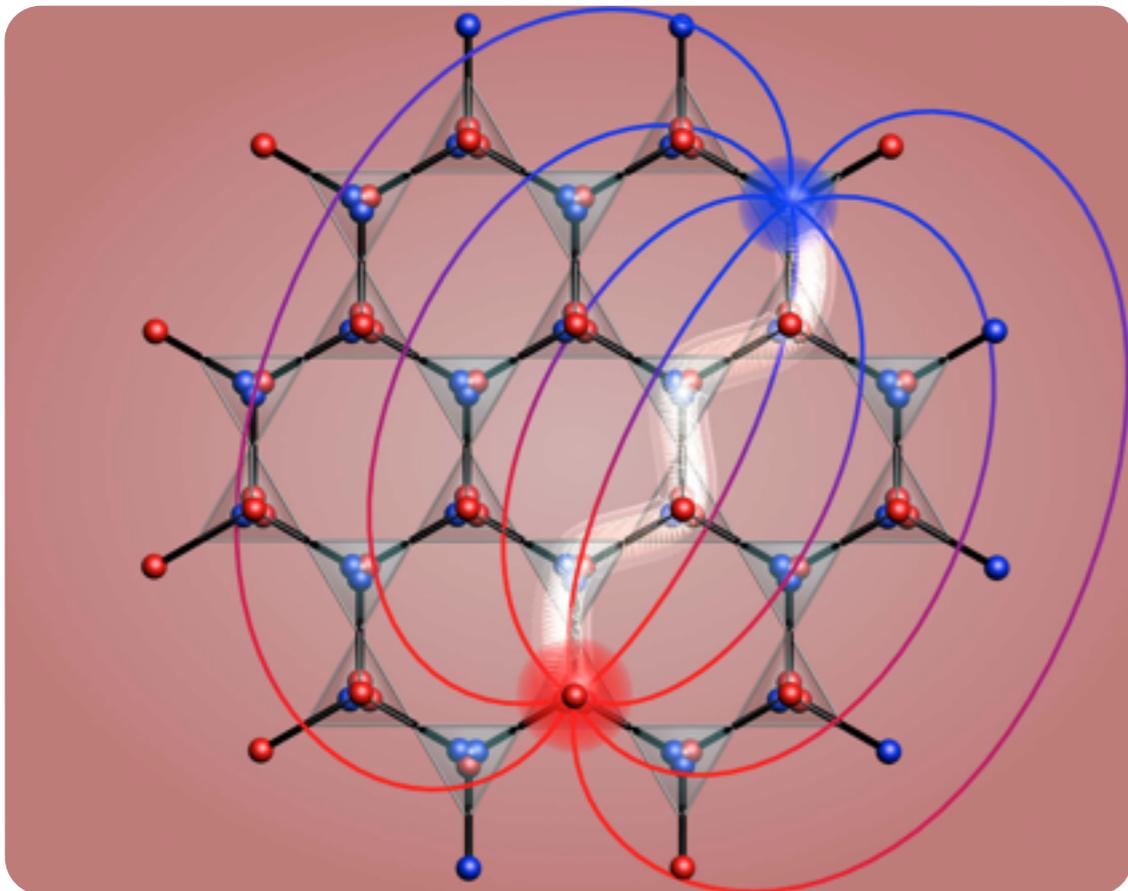
Example: Ising Ferromagnetic Pyrochlore Classical Spin Ice

ferromagnetic Ising exchange

$$H = J_{zz} \sum_{\langle ij \rangle} \vec{S}_{z_i} \cdot \vec{S}_{z_j}$$



Ferromagnetic Ising exchange gives “Ice Rules”: Two-in Two-out

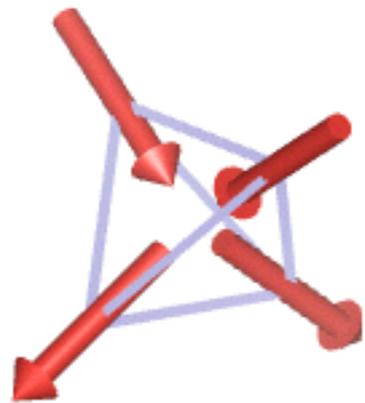


Castelnovo, Moessner, Sondhi. Nature, 451 (2008)

- “Spin ice” chooses between many disordered states obeying 2-in-2-out rules
- Excitations: **deconfined emergent magnetic monopoles**
- **Quantum spin ice:** tunneling between ice like ground states, produces additional emergent excitations

Magnetic Ground States in Pyrochlores (incomplete list!)

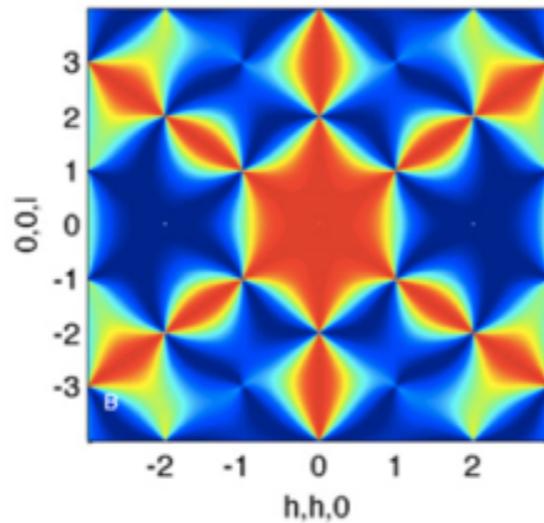
Spin Ice



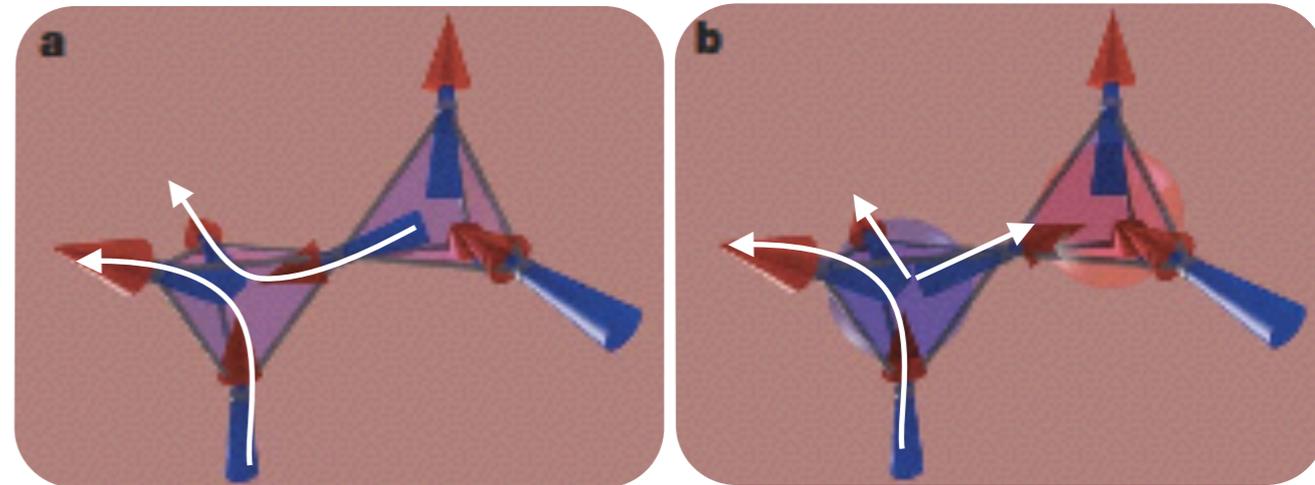
FM Ising



T. Fennell, *Collection SFN 13*, 04001 (2014)



Castelnovo, Moessner, Sondhi. *Nature*, 451 (2008)



Emergent magnetic monopole excitations

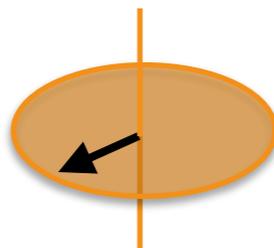
Quantum Spin Liquids

FM Ising

XY



+



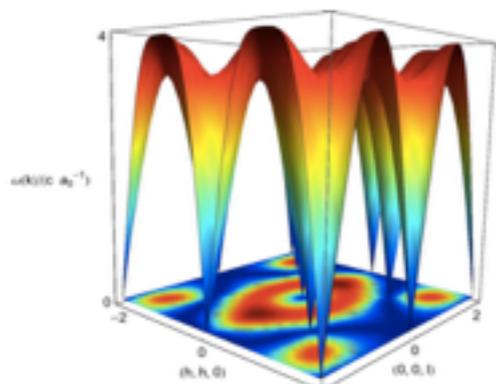
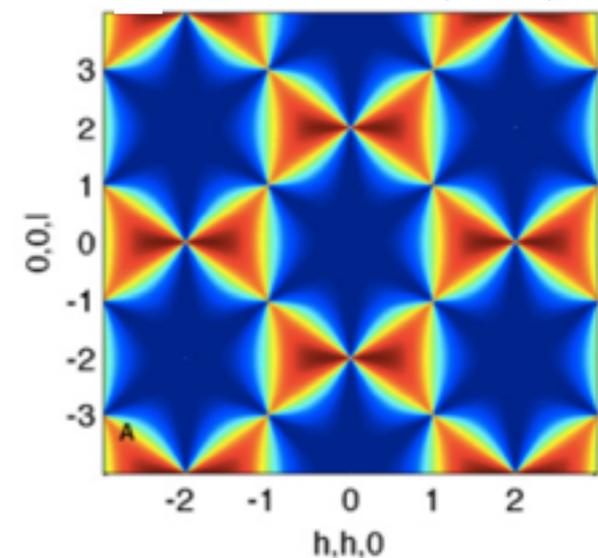
Or

AFM $S=1/2$ Heisenberg



Coulomb phase

T. Fennell, *Collection SFN 13*, 04001 (2014)



“Quantum Spin Ice”

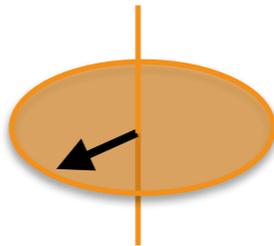
Real Pyrochlores: playgrounds for frustration

$R_2Ti_2O_7$ “Rare earth titanates”

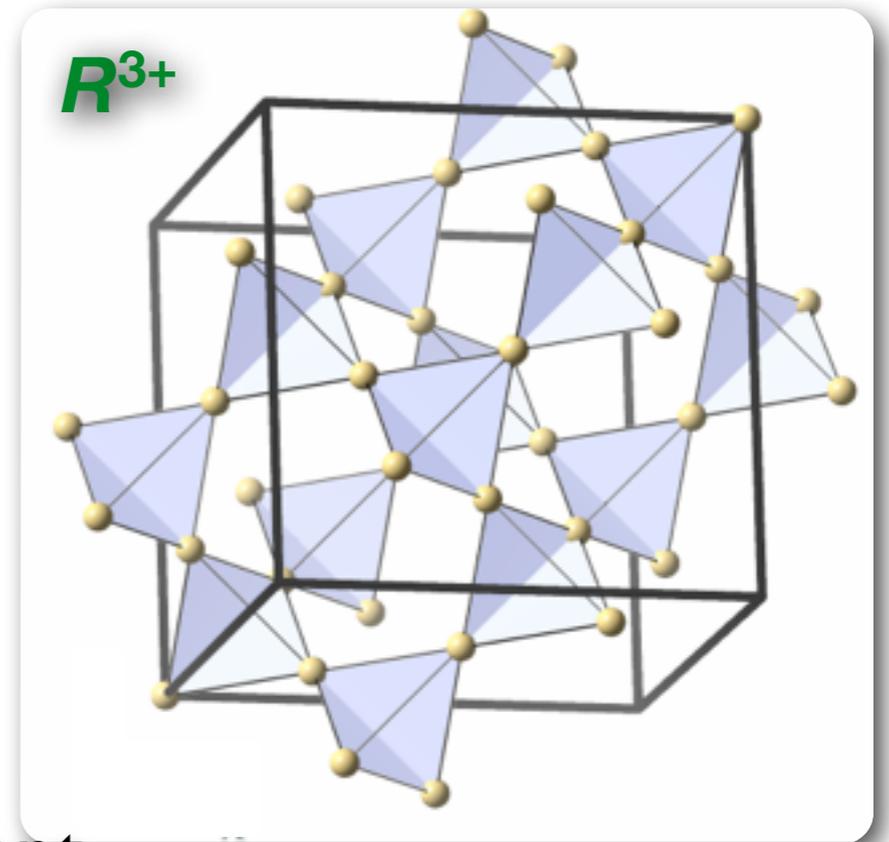
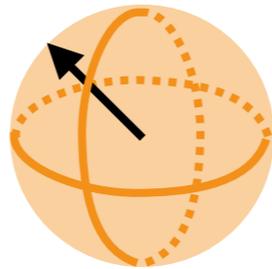
Ising



XY



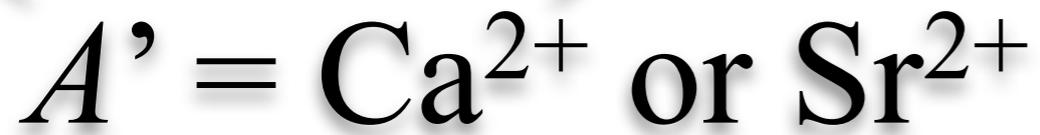
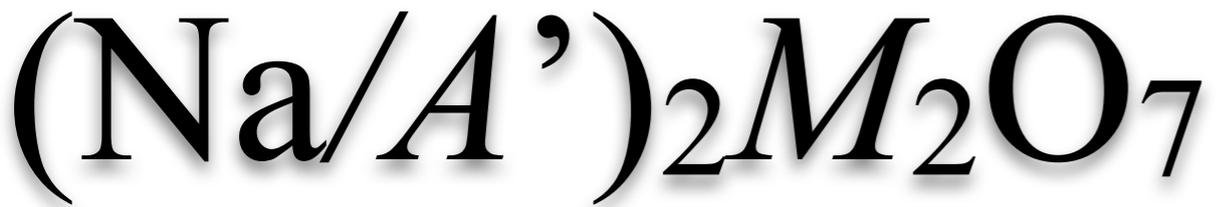
Heisenberg



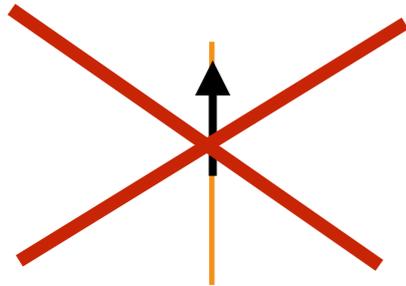
Differences in **single ion anisotropy** is very important

| | Single Ion Anisotropy | Interactions | Ground state |
|--------|-----------------------|--------------|--|
| Ho, Dy | Ising | FM | spin ice |
| Tb | Ising | AFM | spin liquid |
| Gd | Heisenberg | AFM | partial order |
| Er | XY | AFM | “order by disorder” |
| Yb | XY | FM | “quantum spin ice”? Phase competition |

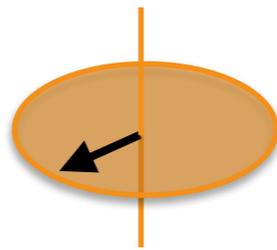
Real Pyrochlores: playgrounds for frustration



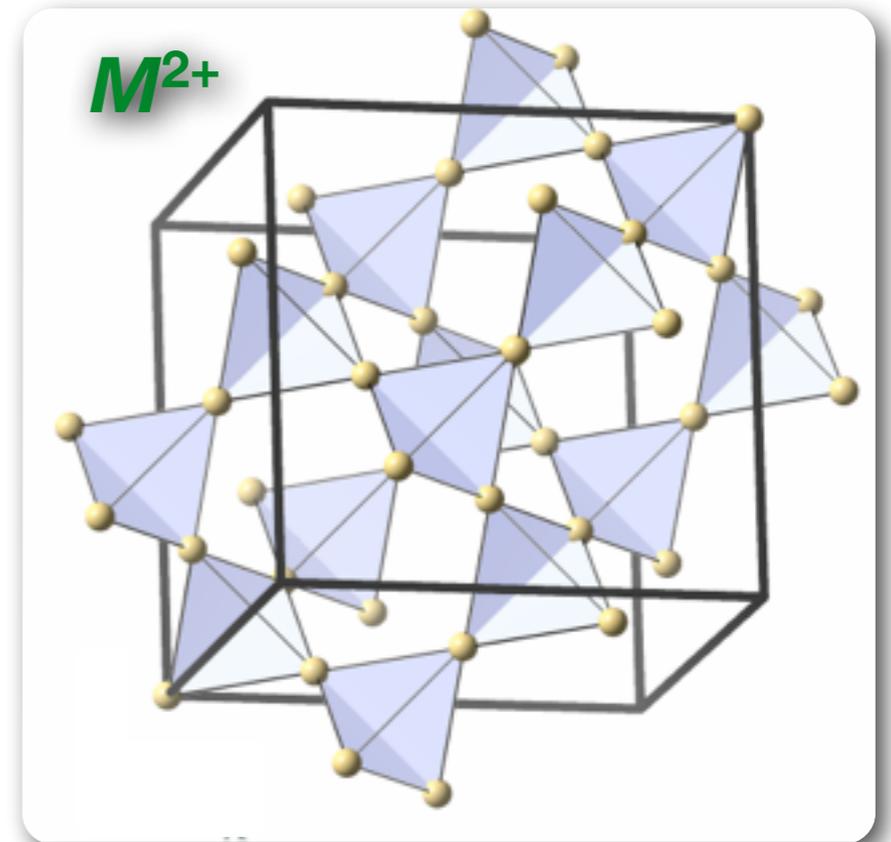
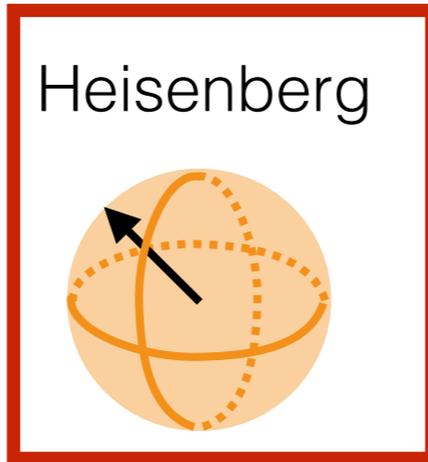
Ising



XY



Heisenberg



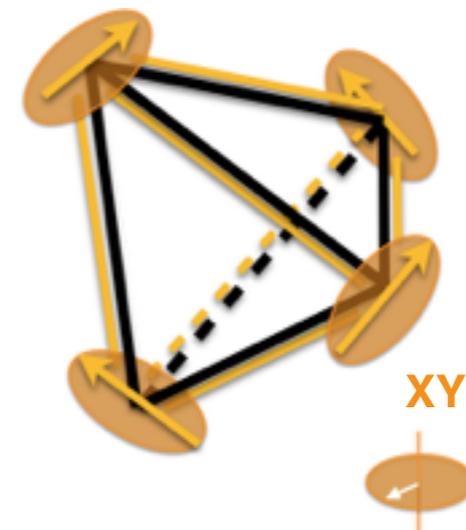
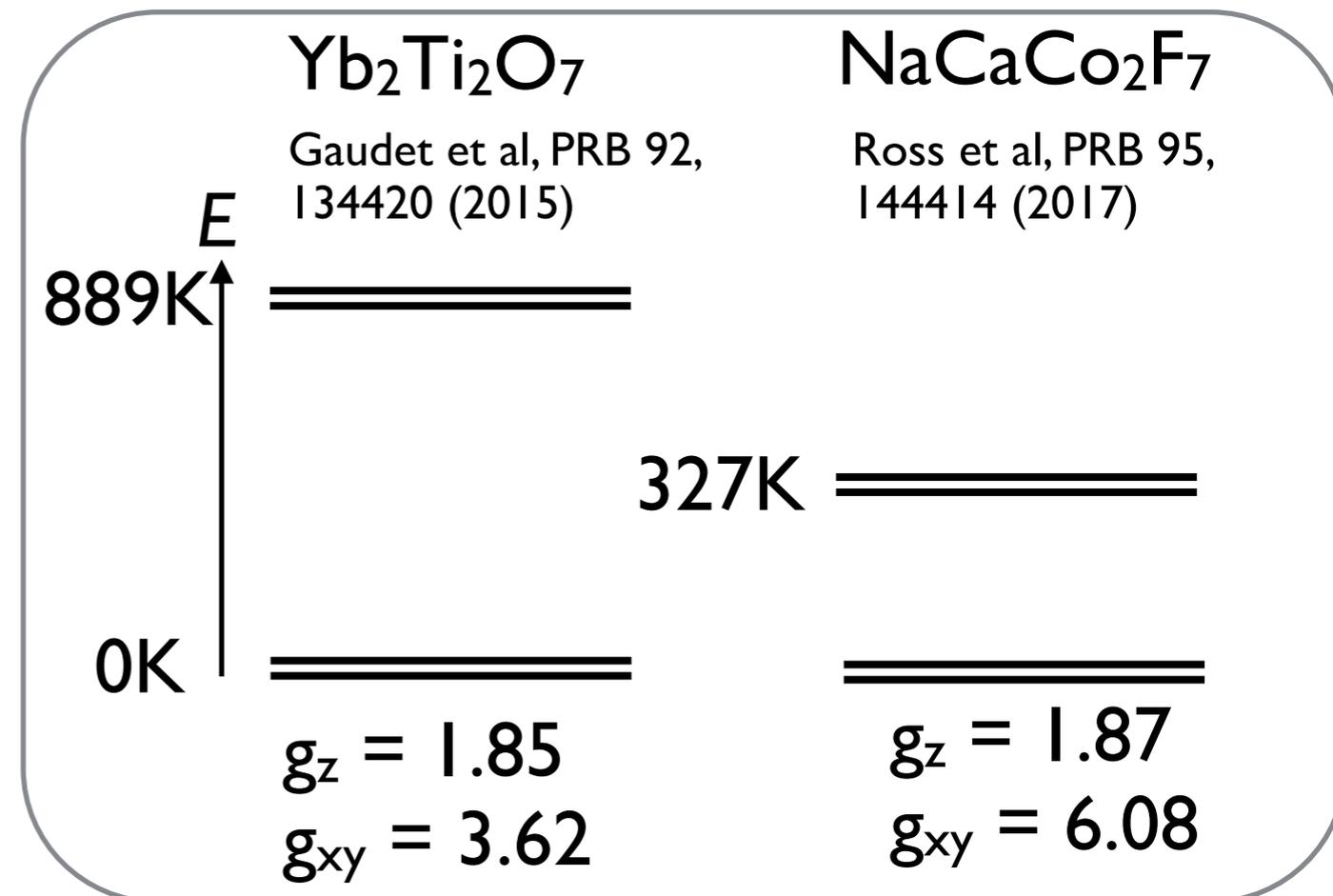
In 3d transition metals, usually Heisenberg — except Co^{2+}

| M^{2+} | Single Ion Anisotropy | Interactions | Ground state |
|-----------|--------------------------|--------------|--|
| Co | XY, $S_{\text{eff}}=1/2$ | AFM | spin frozen only at low effective T |
| Mn | Heisenberg? $S=5/2$ | AFM | spin frozen |
| Ni | Heisenberg? $S=1$ | AFM | spin frozen |
| Fe | Heisenberg? $S=5/2$ | AFM | spin frozen |

Origin of anisotropy in Pyrochlores

$R_2B_2O_7$ (R^{3+} = rare earth, B^{4+} = Ti, Sn, Pt, Ge),
NaCa**Co** $_2$ F $_7$, NaSr**Co** $_2$ F $_7$

- **Spin orbit coupling plus crystal field** in both Co^{2+} and R^{3+} : J instead of S and L
- **Effective $S=1/2$ doublets**
- **g-tensor**: describes size of single-ion magnetic moment in various directions
- **anisotropic exchange couplings**: Distinct from g-tensor anisotropy!



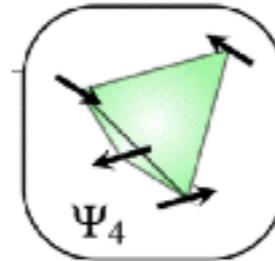
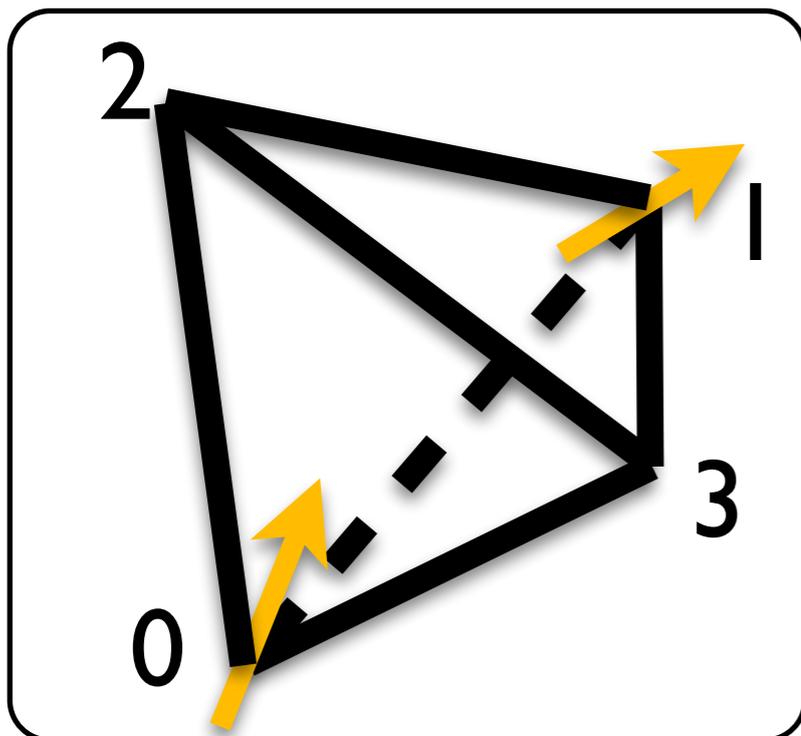
General Anisotropic Exchange

Tensor with 4 unique elements:

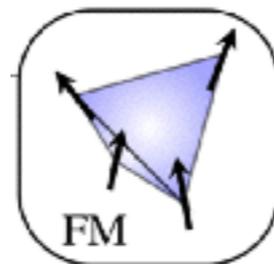
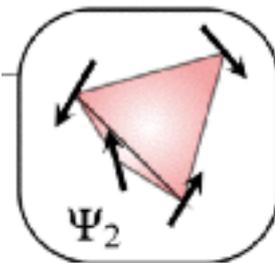
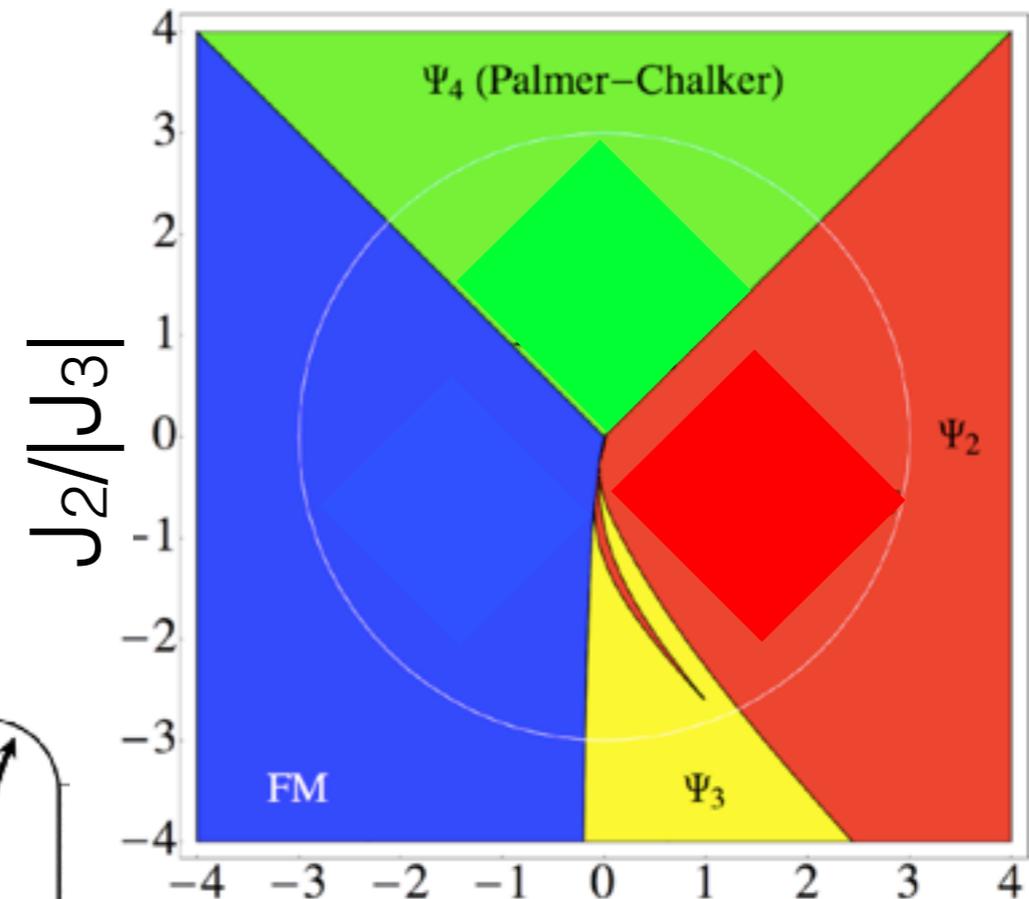
$$J_2 S_1^x S_2^x + J_1 (S_1^y S_2^y + S_1^z S_2^z) + J_3 (S_1^y S_2^z + S_1^z S_2^y) + \dots$$

$$H = \frac{1}{2} \sum_{ij} J_{ij}^{\mu\nu} S_i^\mu S_j^\nu$$

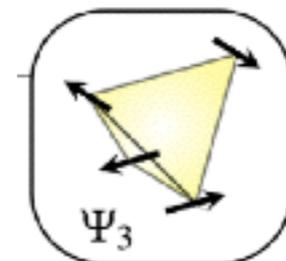
$$J_{01} = \begin{pmatrix} J_2 & J_4 & J_4 \\ -J_4 & J_1 & J_3 \\ -J_4 & J_3 & J_1 \end{pmatrix}$$



**Classical phase diagram:
Types of Long Range
Order (for $J_4 = 0$)**



$J_1/|J_3|$

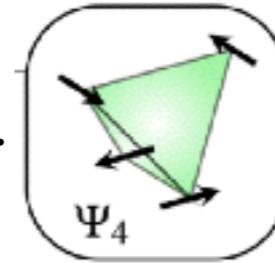


Yan et al, "General theory of anisotropic exchange on the pyrochlore lattice", PRB **95**, 094422 (2017)

General Anisotropic Exchange

Tensor with 4 unique elements:

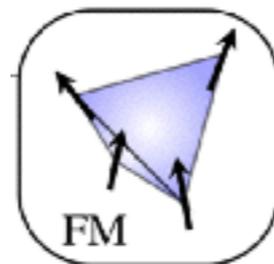
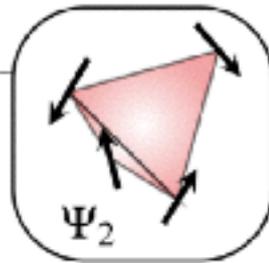
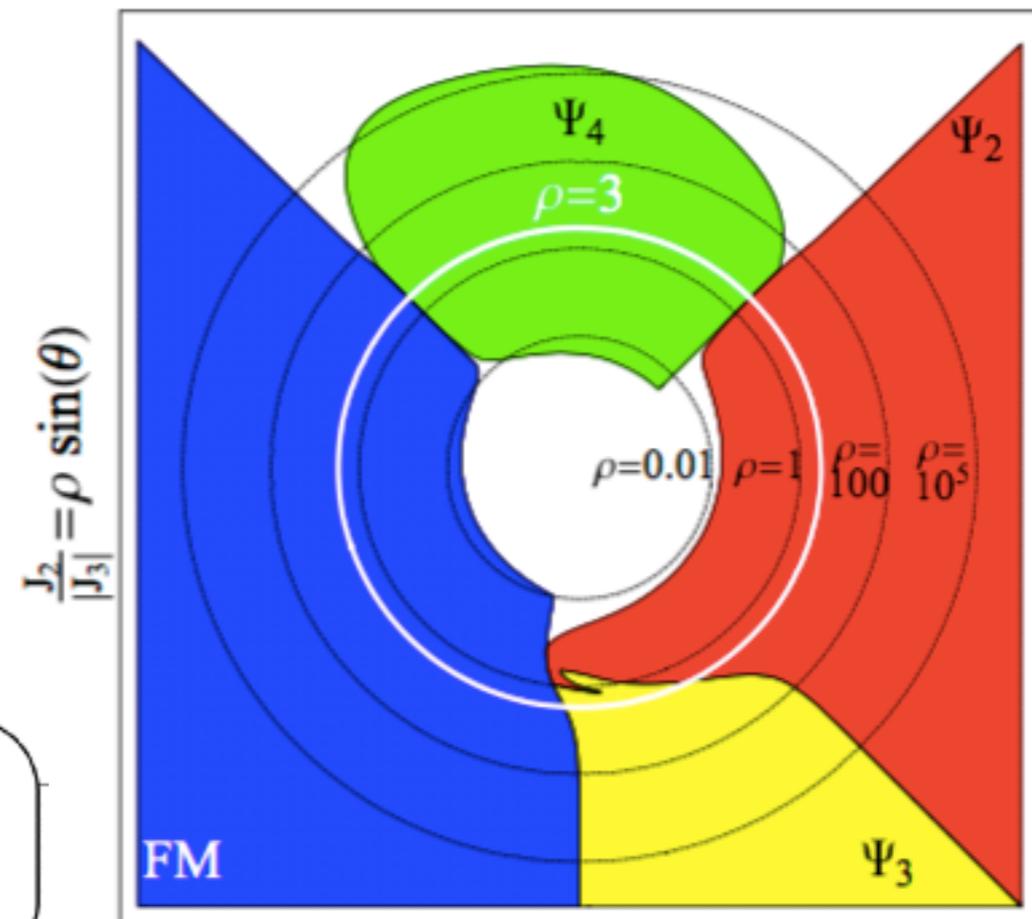
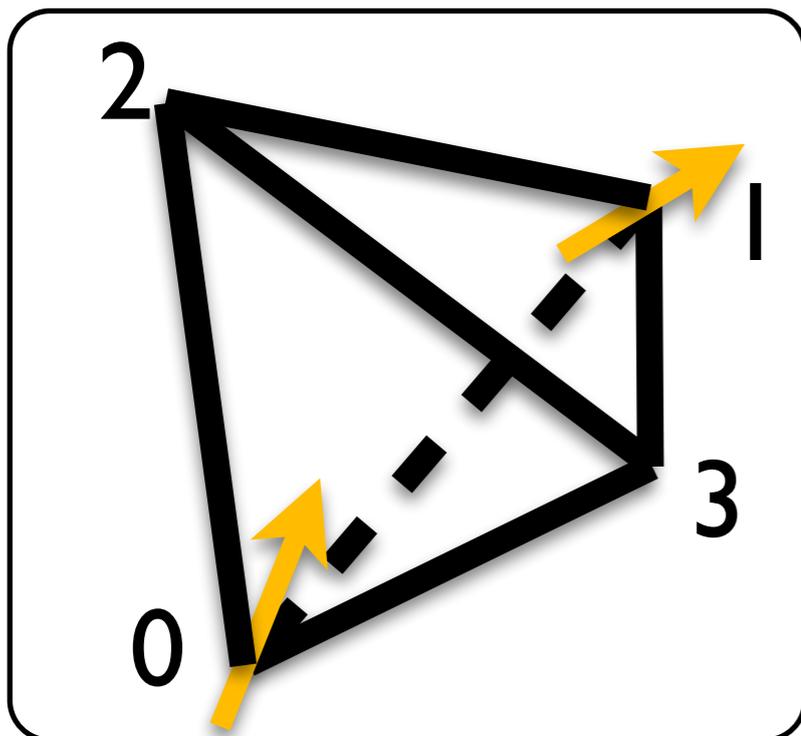
$$J_2 S_1^x S_2^x + J_1 (S_1^y S_2^y + S_1^z S_2^z) + J_3 (S_1^y S_2^z + S_1^z S_2^y) + \dots$$



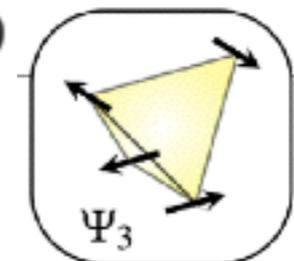
**White regions:
quantum fluctuations
destroy conventional
LRO**

$$H = \frac{1}{2} \sum_{ij} J_{ij}^{\mu\nu} S_i^\mu S_j^\nu$$

$$J_{01} = \begin{pmatrix} J_2 & J_4 & J_4 \\ -J_4 & J_1 & J_3 \\ -J_4 & J_3 & J_1 \end{pmatrix}$$



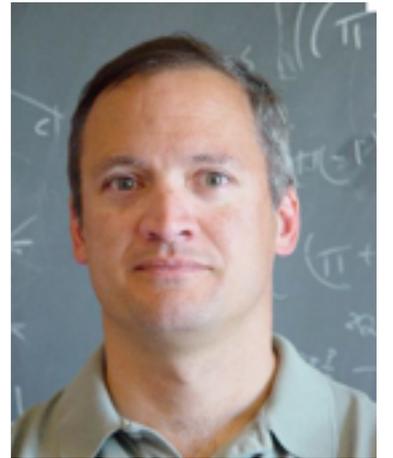
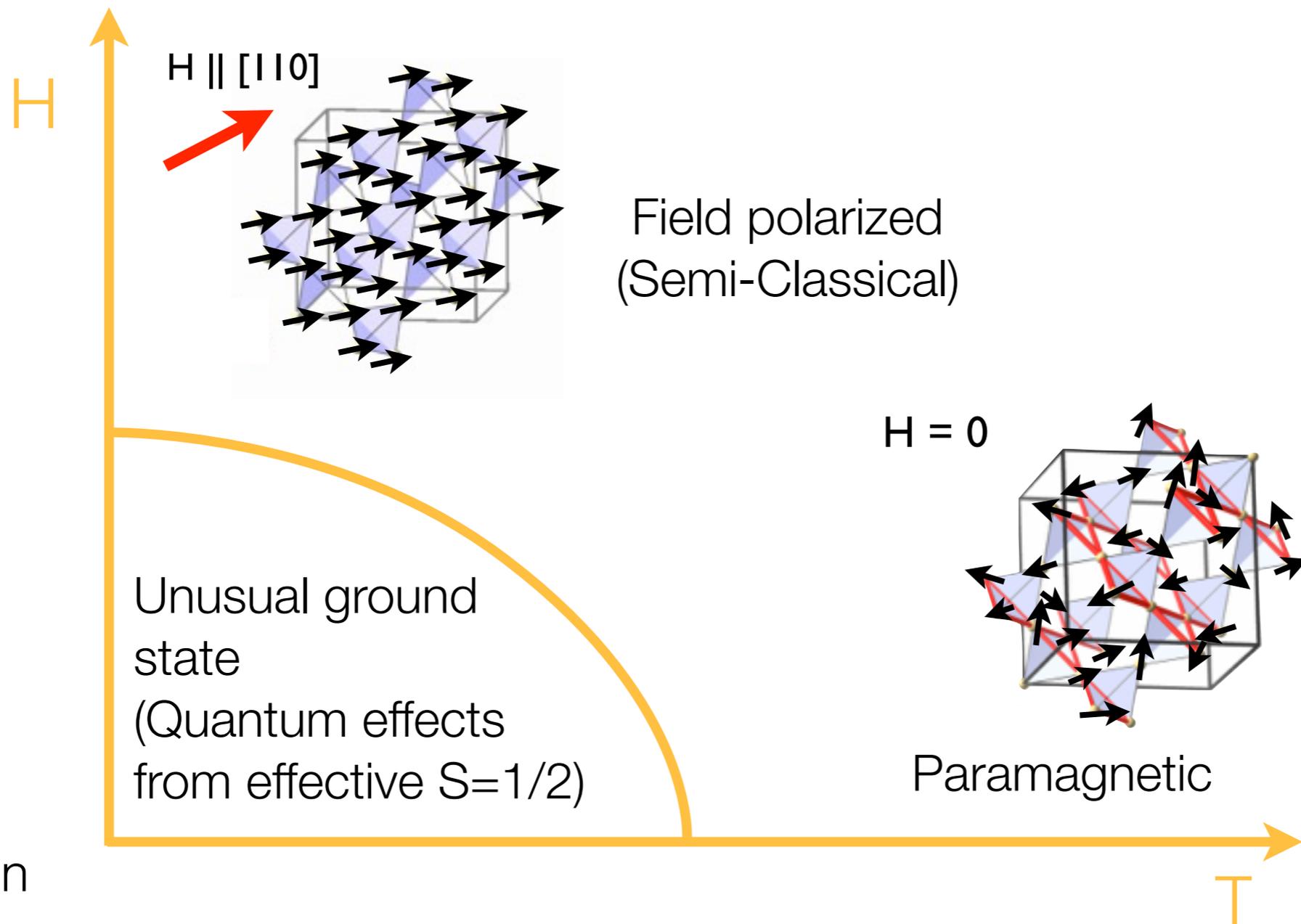
$$\frac{J_1}{|J_3|} = \rho \cos(\theta)$$



Yan et al, "General theory of anisotropic exchange on the pyrochlore lattice", PRB **95**, 094422 (2017)

Determining exchange interactions from field polarized states

Can extract quantitative values for J_1 - J_4
Linear Spin Wave Theory + Neutron Scattering



Leon Balents



Bruce Gaulin

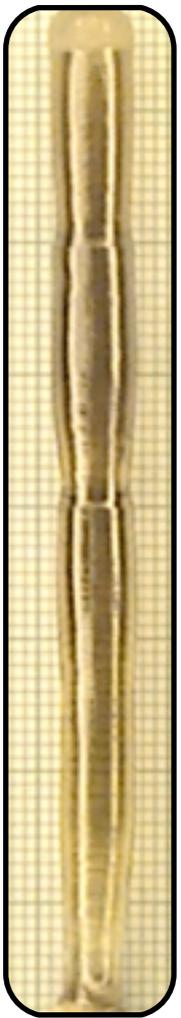


Lucile Savary

“Time of Flight” Inelastic Neutron Scattering

“Disk Chopper Spectrometer”
(DCS)

@ NIST Center for
Neutron Research



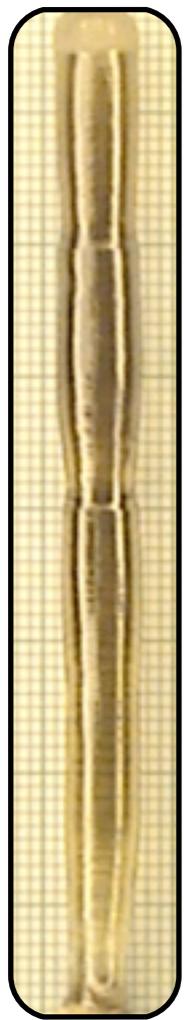
Single Crystal
 $\text{Yb}_2\text{Ti}_2\text{O}_7$

7.5 cm

“Time of Flight” Inelastic Neutron Scattering

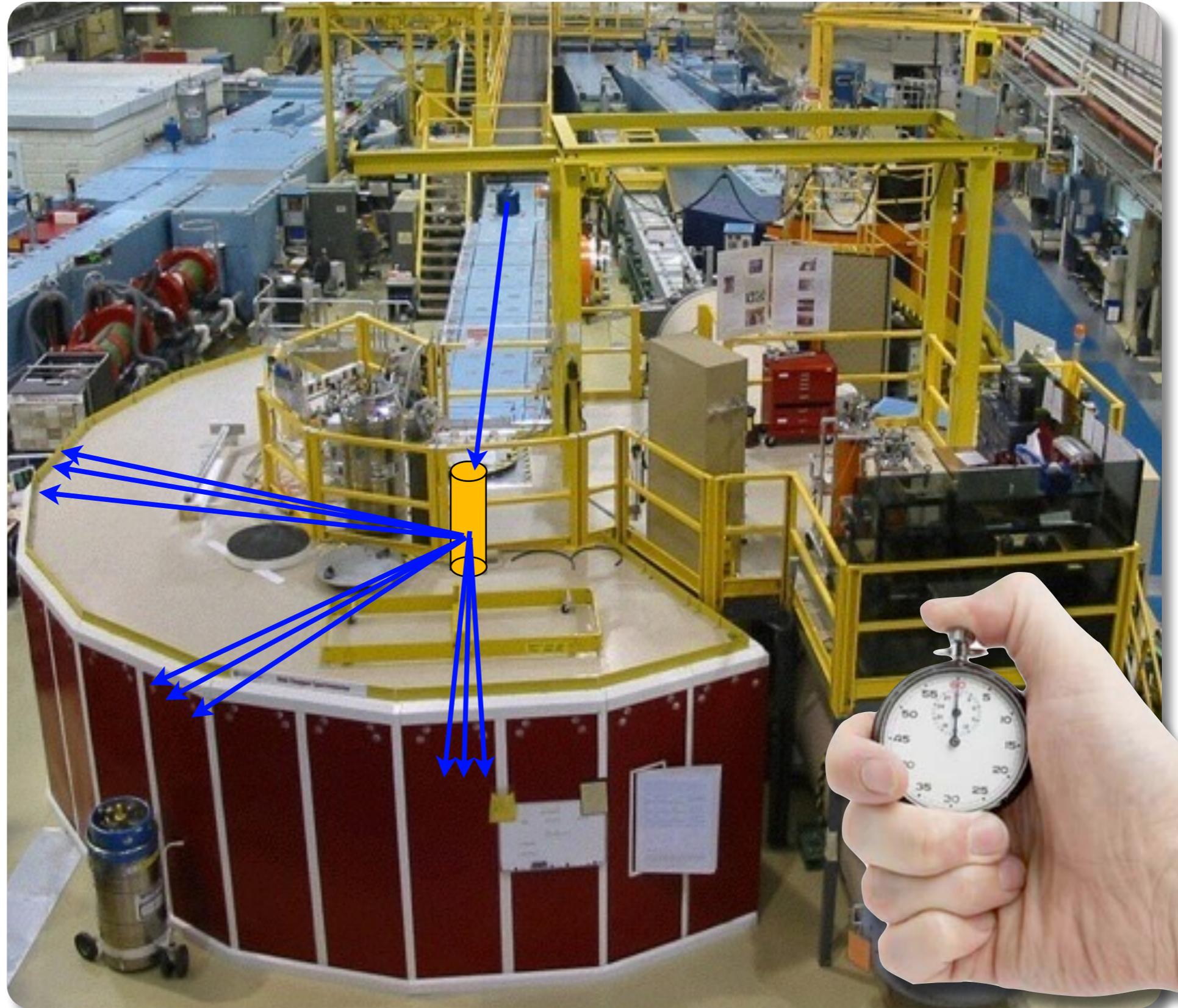
“Disk Chopper Spectrometer”
(DCS)

@ NIST Center for
Neutron Research



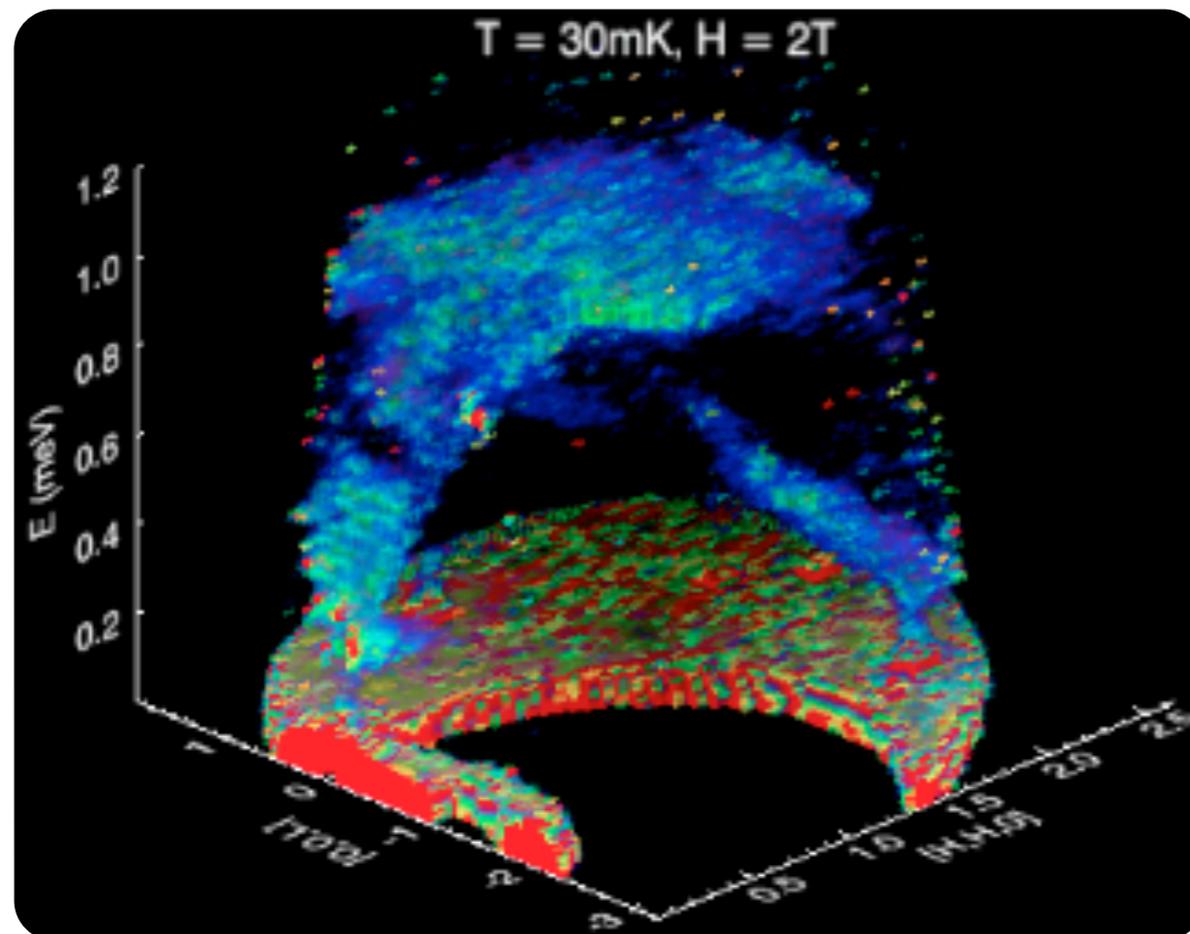
Single Crystal
 $\text{Yb}_2\text{Ti}_2\text{O}_7$

7.5 cm



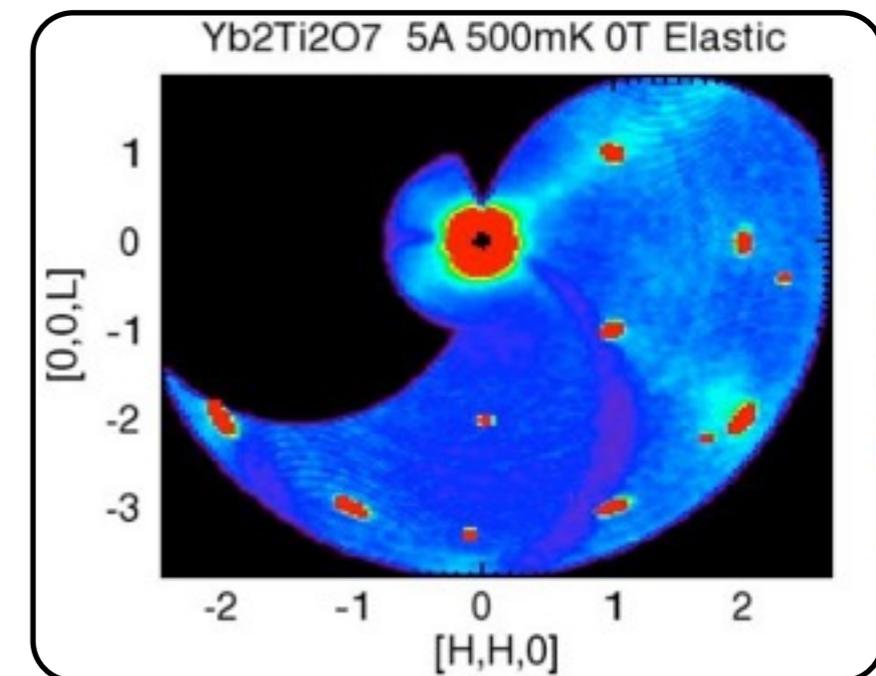
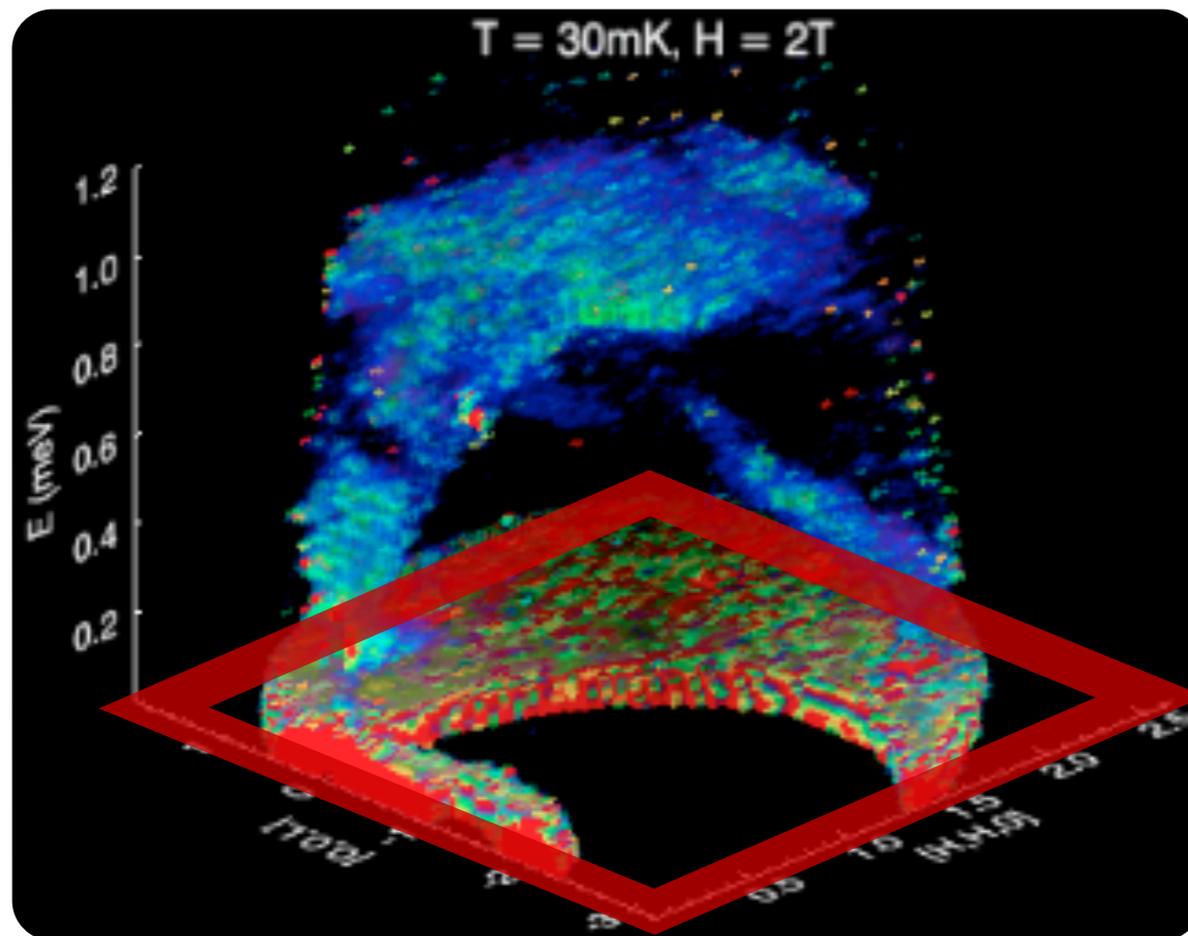
Volume of “Time of Flight” Data

Can slice through this volume in several directions



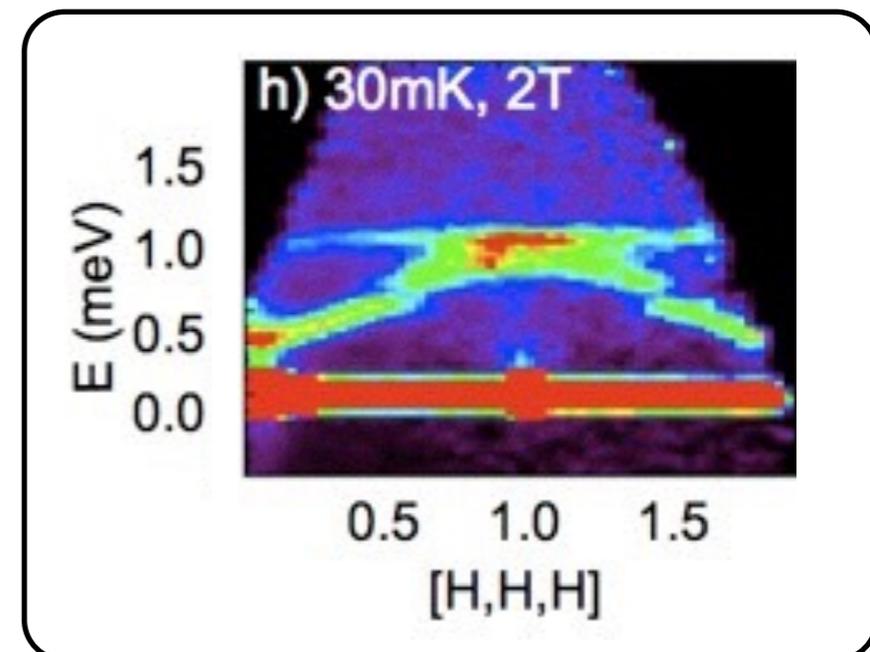
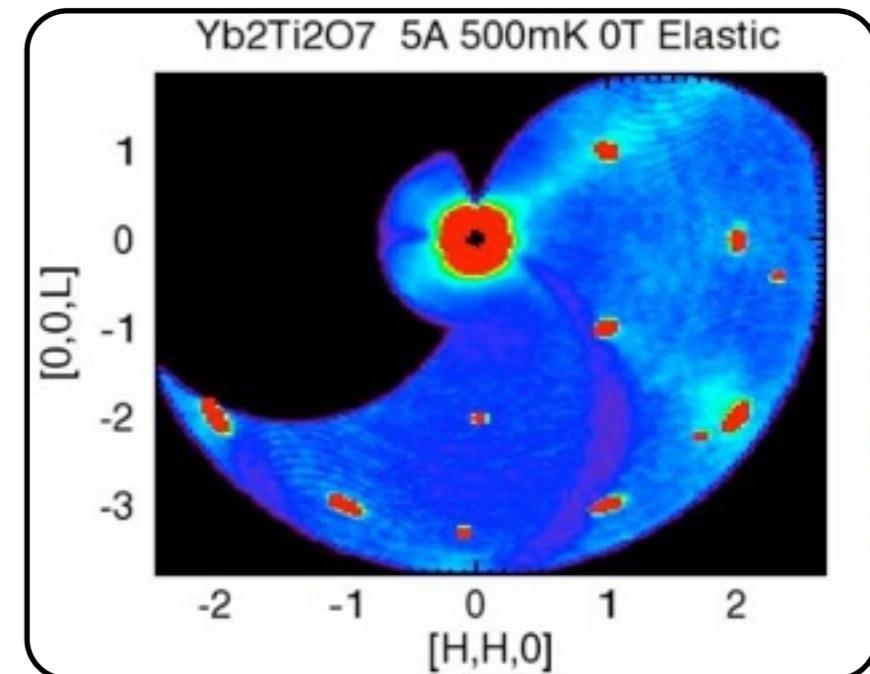
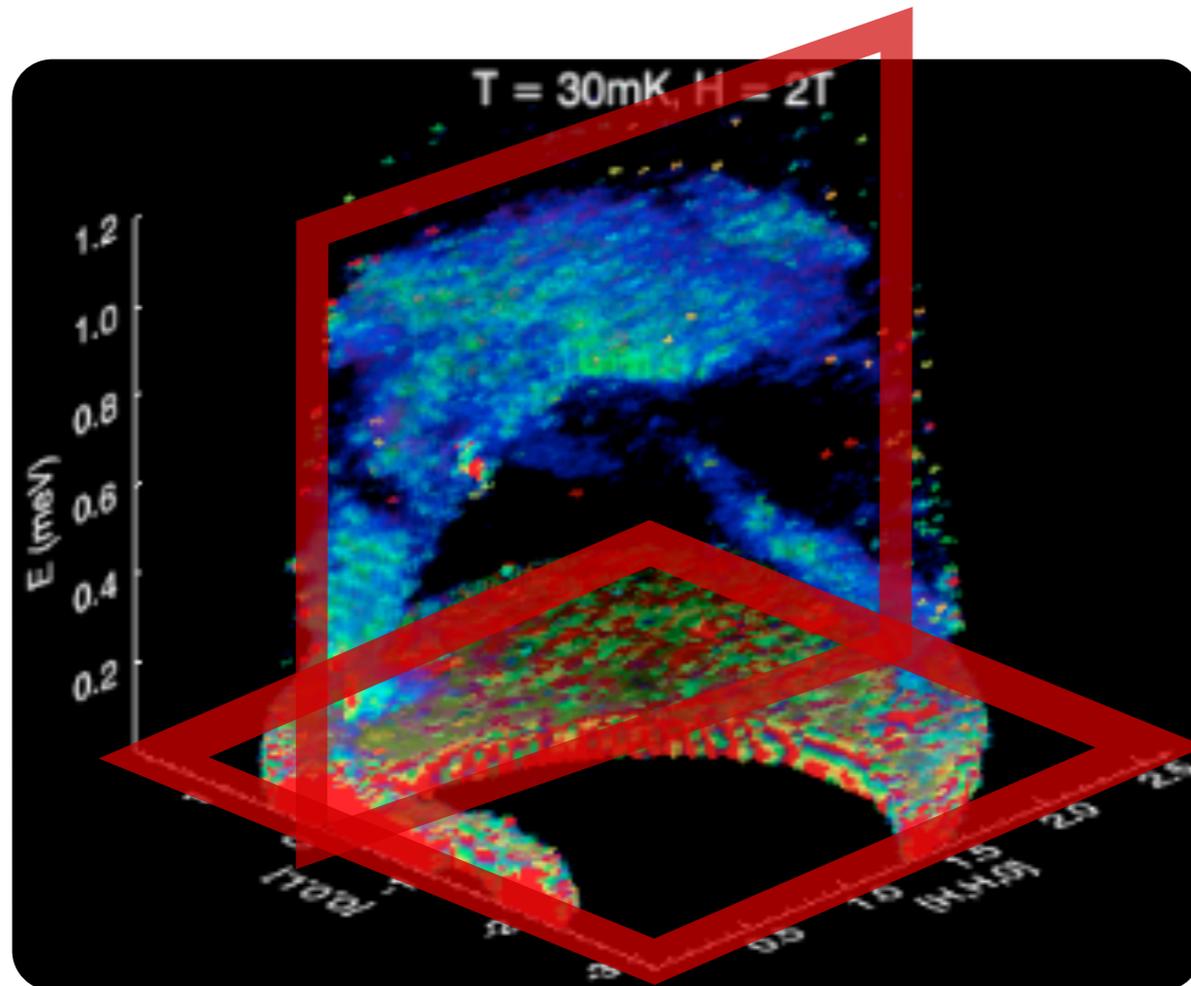
Volume of “Time of Flight” Data

Can slice through this volume in several directions



Volume of “Time of Flight” Data

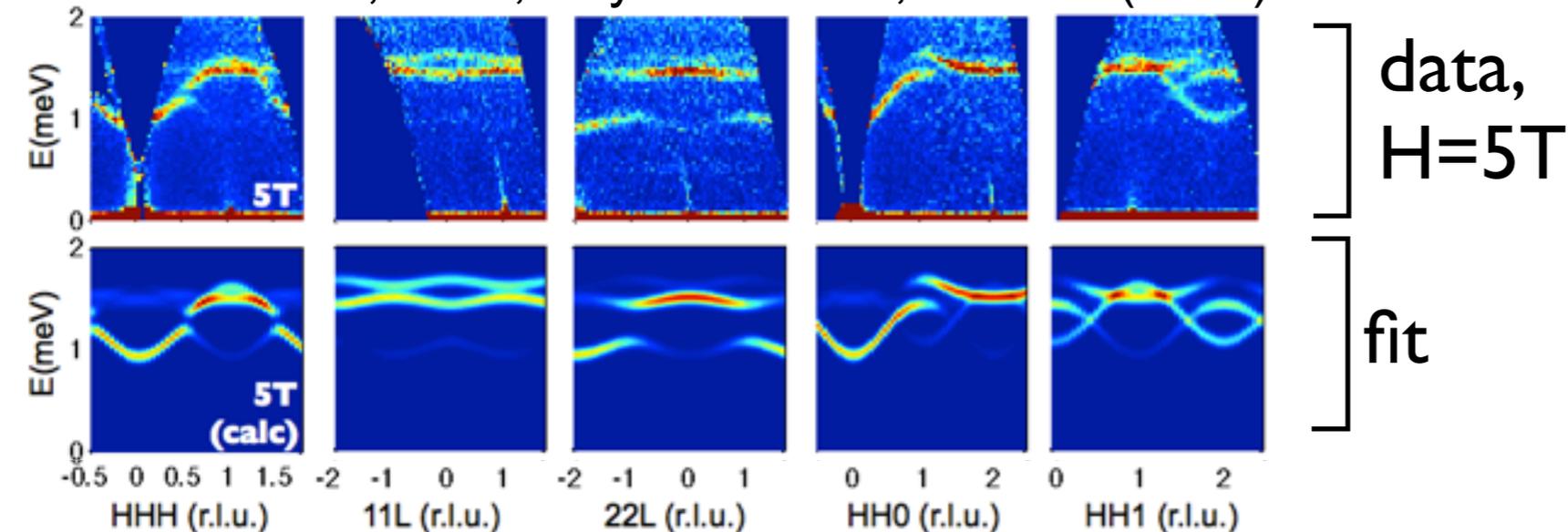
Can slice through this volume in several directions



Determining exchange interactions with spin waves from field polarized state

Yb₂Ti₂O₇

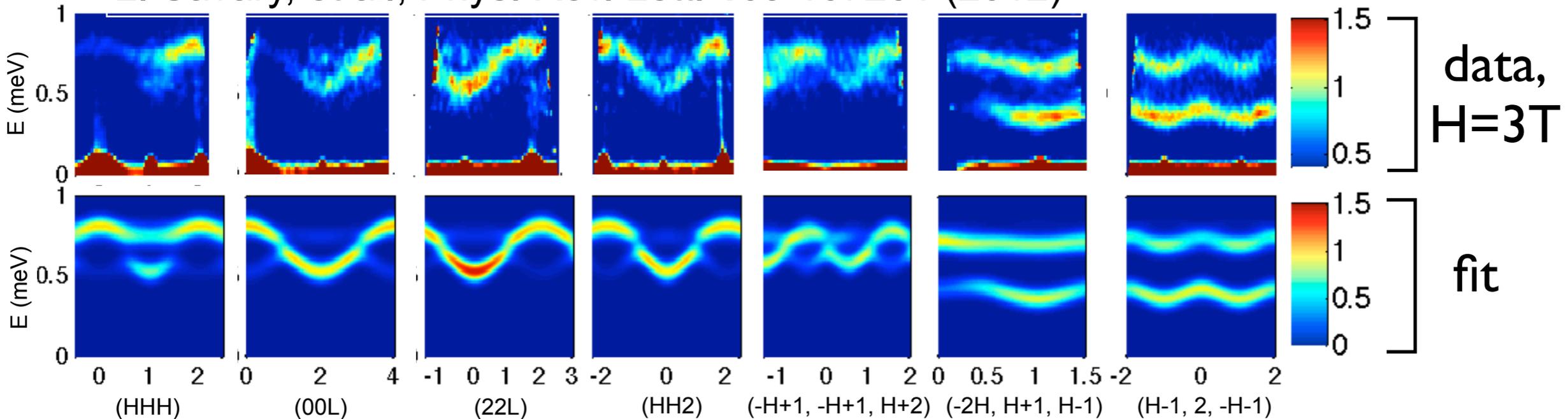
K.A. Ross, et al., Phys. Rev. X **1**, 021002 (2011)



| | Yb ₂ Ti ₂ O ₇ | Er ₂ Ti ₂ O ₇ |
|----|--|--|
| J1 | -0.09 | 0.11 |
| J2 | -0.22 | -0.06 |
| J3 | -0.29 | -0.10 |
| J4 | 0.01 | 0.00 |

Er₂Ti₂O₇

L. Savary, et al., Phys. Rev. Lett. **109** 167201 (2012)



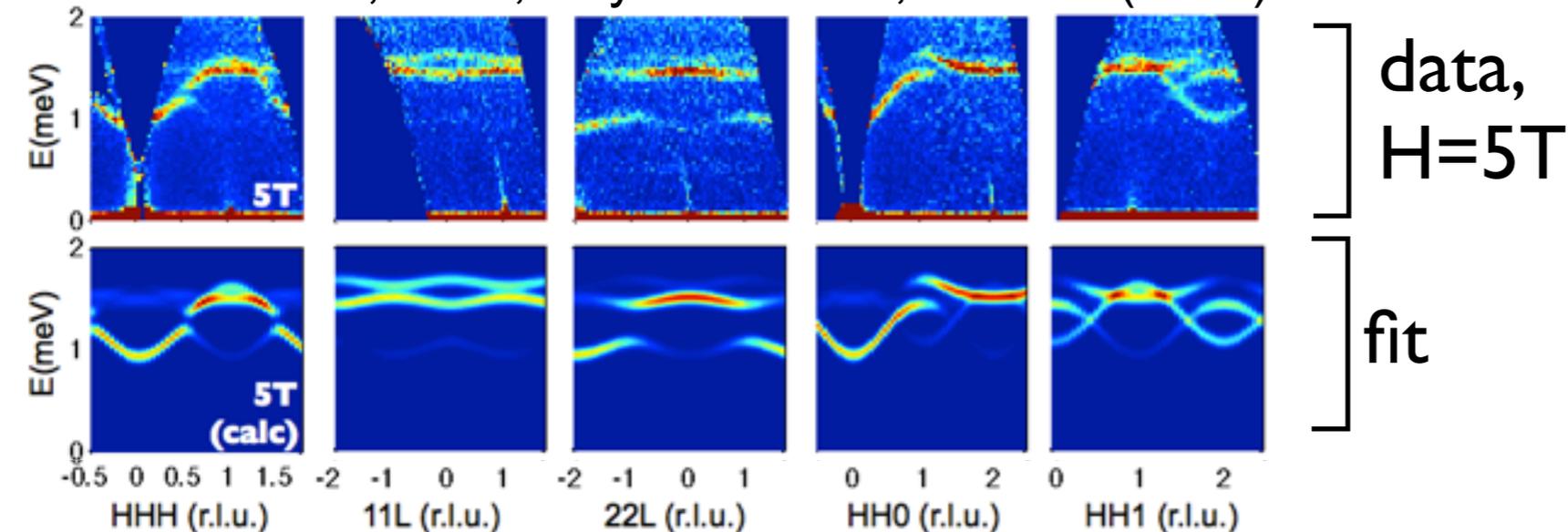
H || to [1-10]

H || to [111]

Determining exchange interactions with spin waves from field polarized state

Yb₂Ti₂O₇

K.A. Ross, et al., Phys. Rev. X **1**, 021002 (2011)

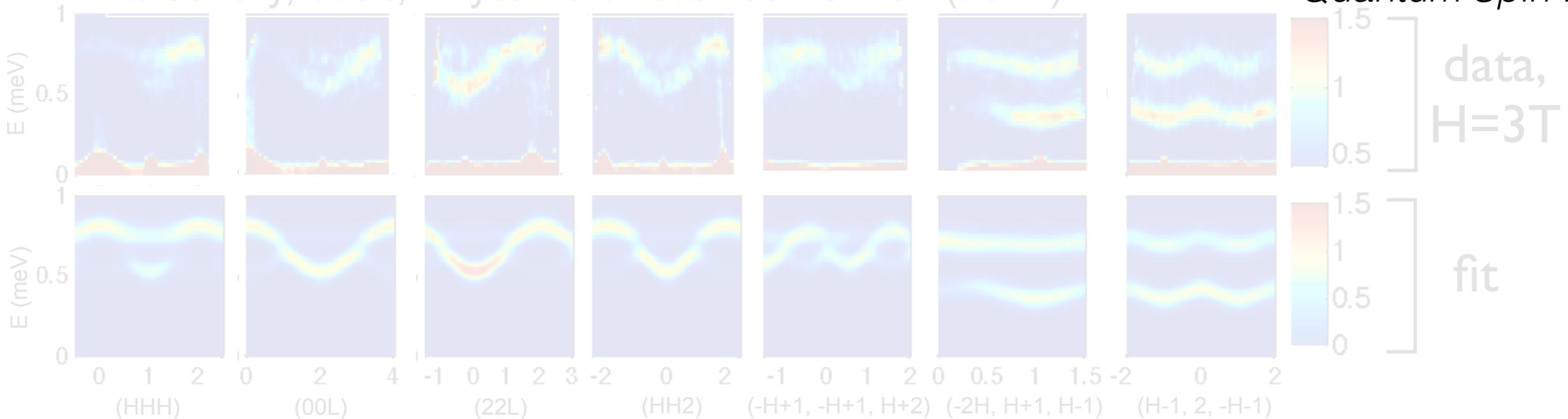


| | Yb ₂ Ti ₂ O ₇ | Er ₂ Ti ₂ O ₇ |
|----|--|--|
| J1 | -0.09 | 0.11 |
| J2 | -0.22 | -0.06 |
| J3 | -0.29 | -0.10 |
| J4 | 0.01 | 0.00 |

Params lead to phase competition, and possible *Quantum Spin Ice*

Er₂Ti₂O₇

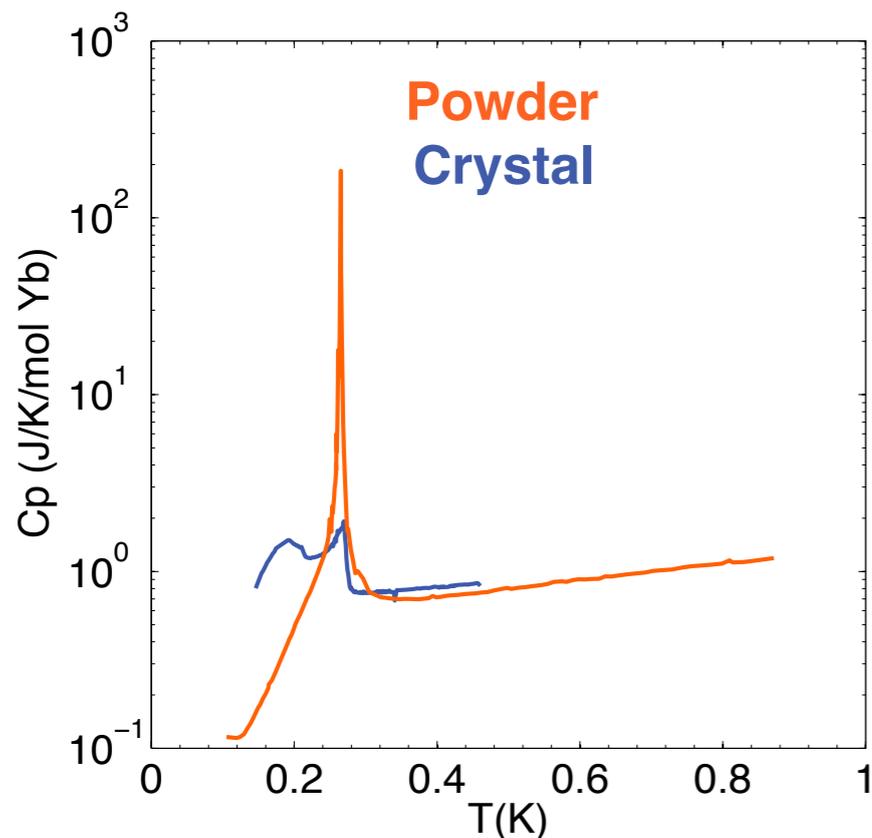
L. Savary, et al., Phys. Rev. Lett. **109** 167201 (2012)



H || to [1-10]

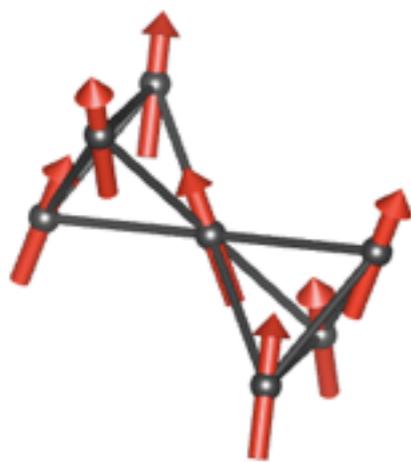
H || to [111]

Yb₂Ti₂O₇: splayed ferromagnet with gapless “continuum” excitations

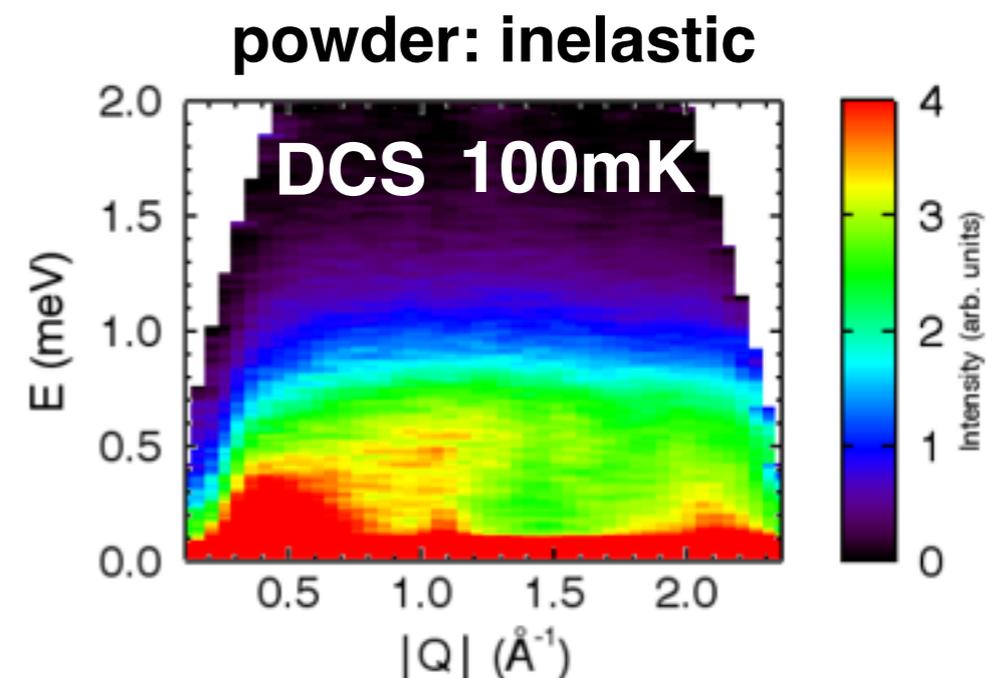
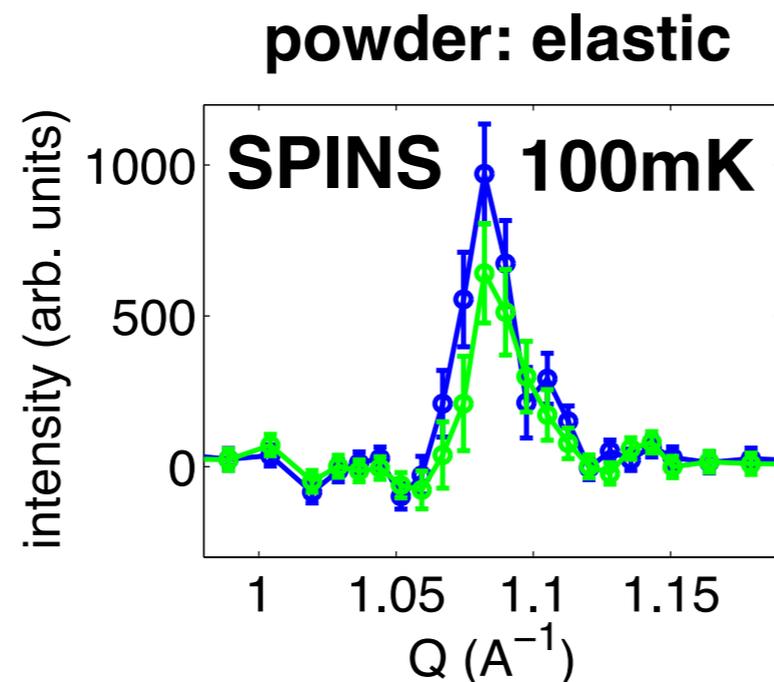


- Heat capacity anomaly at low temperatures, with some sample dependence
- “**Best**” samples (usually powders) show Ice-like **splayed ferromagnetic order** at 265 mK
- Despite this, excitations are relatively unstructured below T_c - **unlike conventional magnons**

J. Gaudet, *et al* PRB **93**, 064406 (2015)



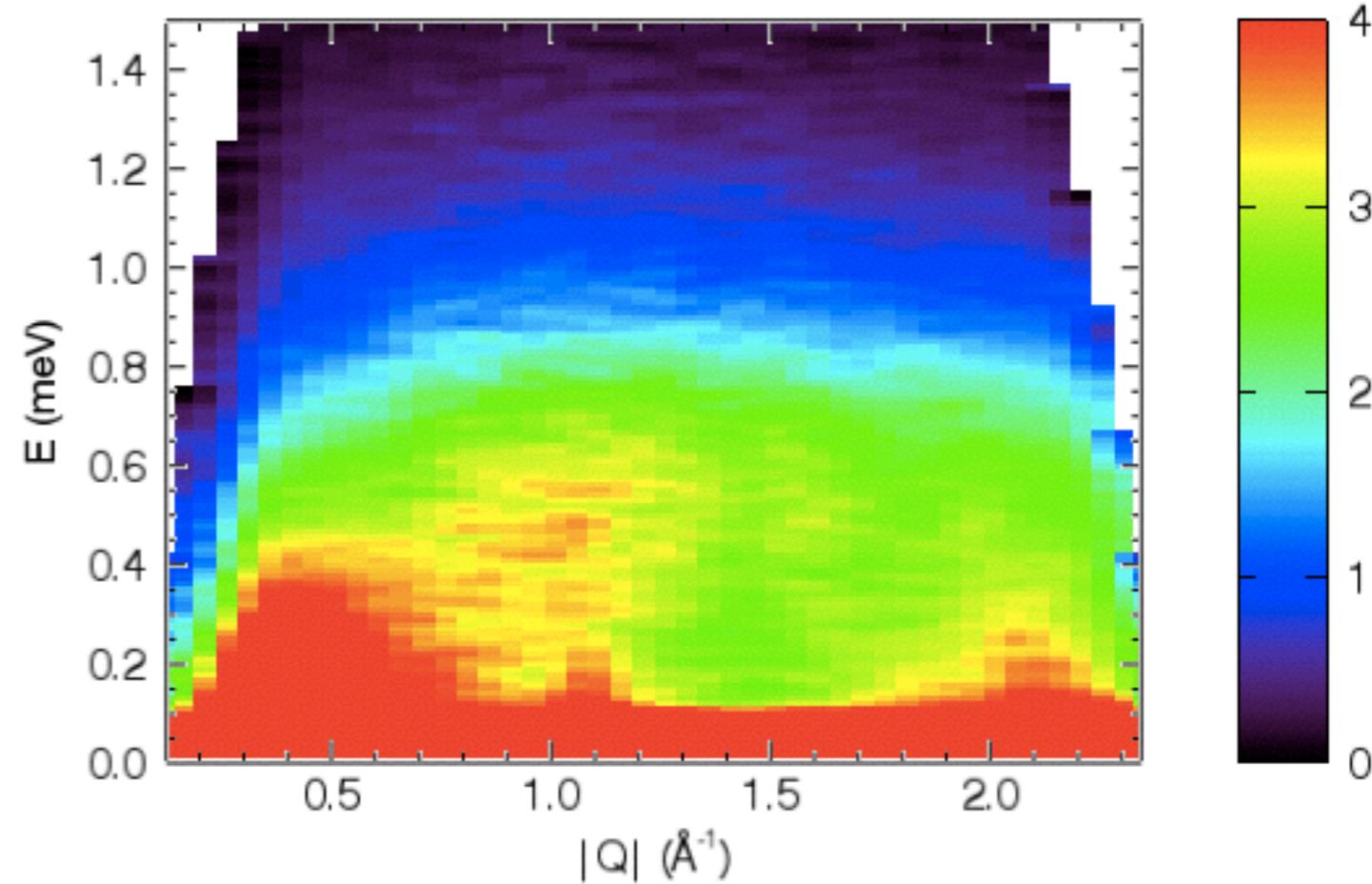
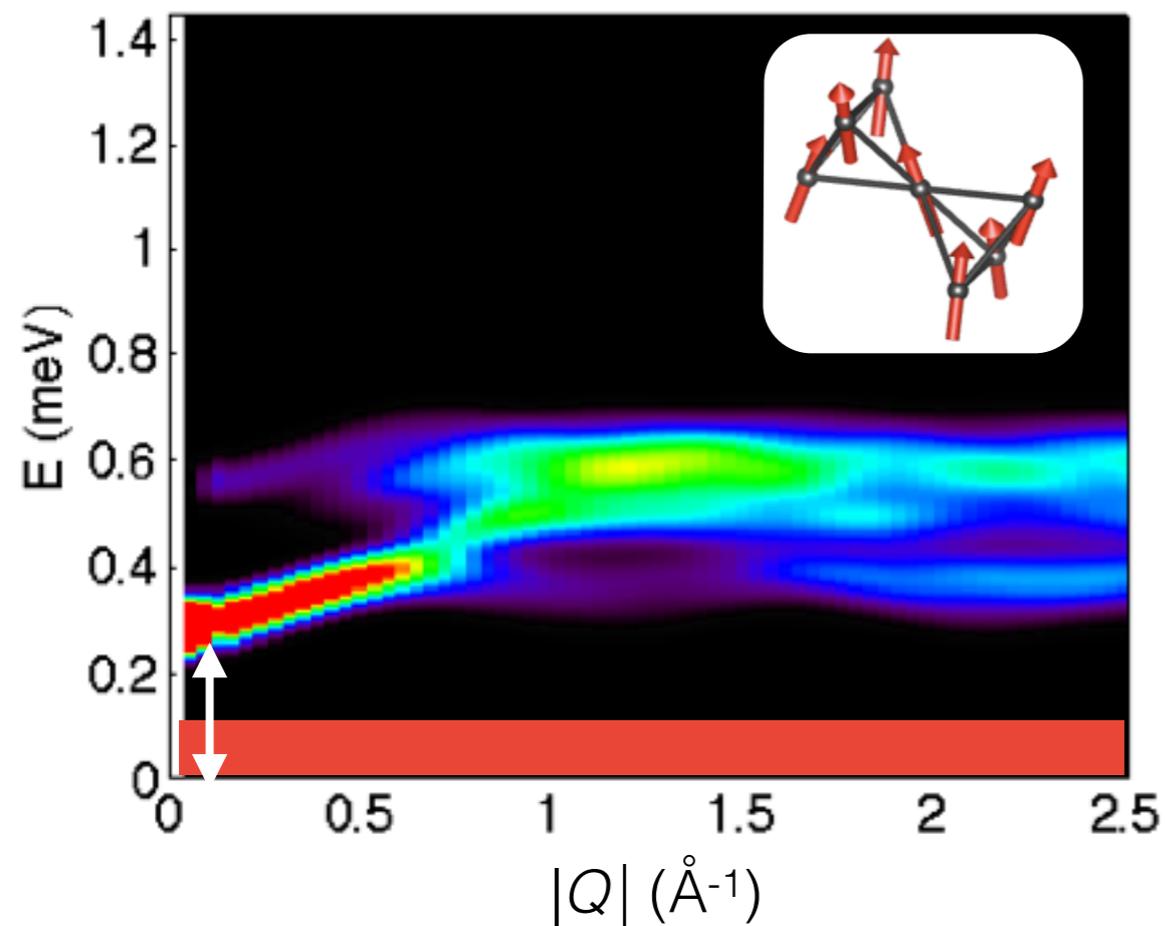
“Spin Ice-Like”
(2-in 2-out) Splaying



Compare zero field spin waves to Linear Spin Wave Theory

Calculated zero-field spin waves

Measured
 $\text{Yb}_2\text{Ti}_2\text{O}_7$ 100 mK

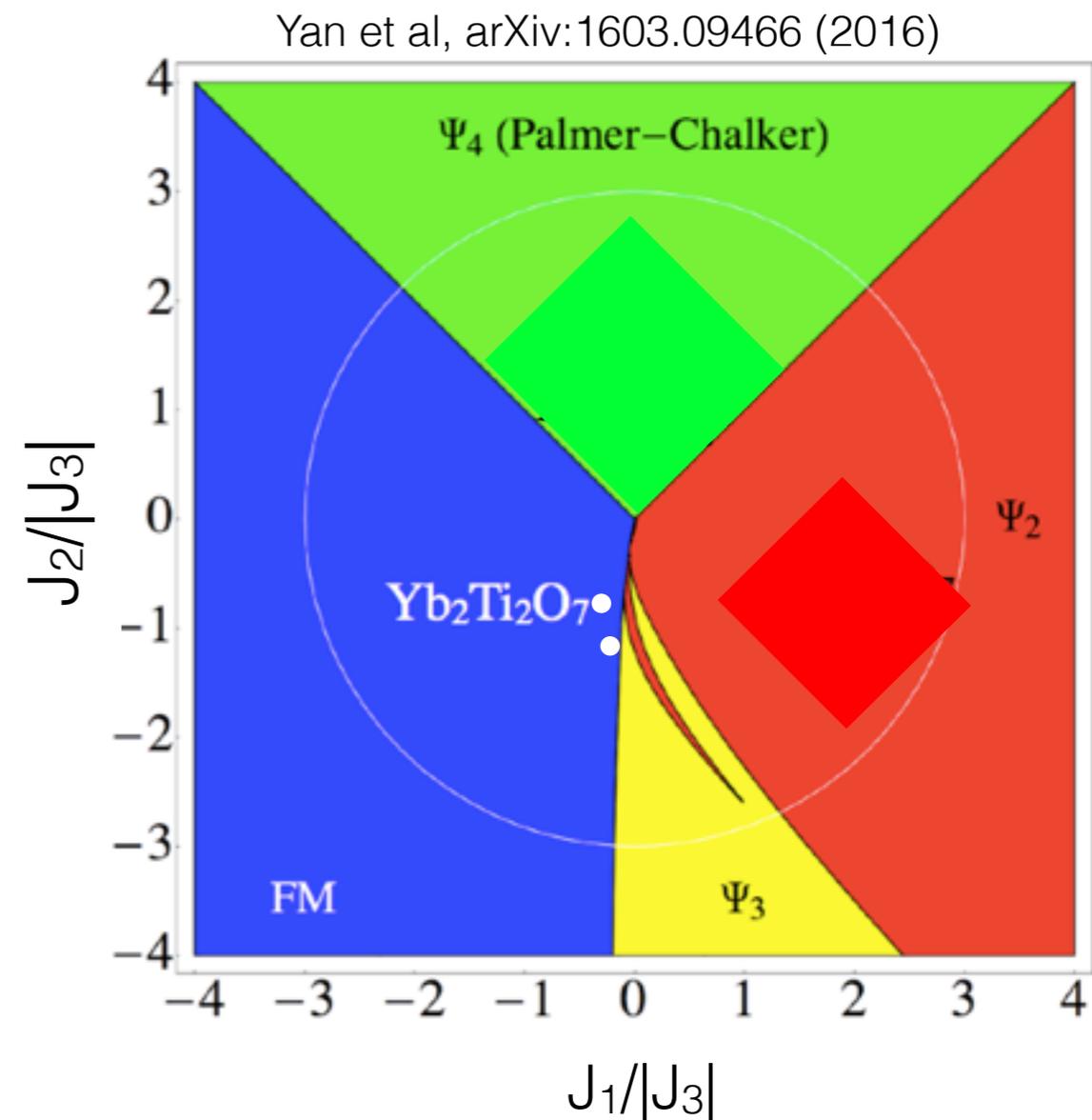


Using Exchange parameters from
Ross *et al*, Phys. Rev. X **1**, 021002 (2011)

Time-of-Flight Spectrometer
(DCS)

Yb₂Ti₂O₇ on phase diagram

- Modified parameters from other groups^[1,2] suggest Yb₂Ti₂O₇ is right on the edge of AFM order
- Do quantum fluctuations arise from proximity to AFM state, i.e. competing orders?
- **What role does the known sample dependence play?**



All proposed parameters put Yb₂Ti₂O₇ close to a classical phase boundary with AFM order

$$J_1 = -0.09, J_2 = -0.22, J_3 = -0.29, J_4 = 0.01$$

K.A. Ross, et al., Phys. Rev. X 1, 021002 (2011)

$$J_1 = -0.03, J_2 = -0.32, J_3 = -0.28, J_4 = 0.02$$

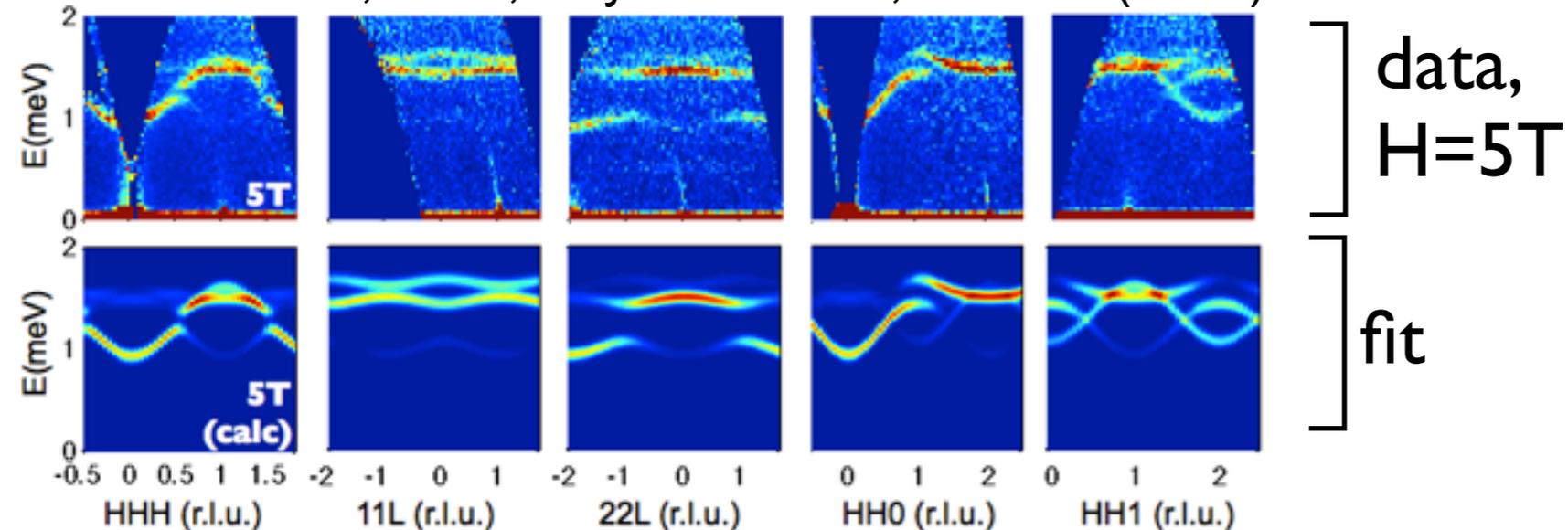
[1] J. Robert, Phys. Rev. B 92, 064425 (2015)

[2] J. Thompson et al, arXiv:1703.04506 [cond-mat.str-el]

Determining exchange interactions with spin waves from field polarized state

Yb₂Ti₂O₇

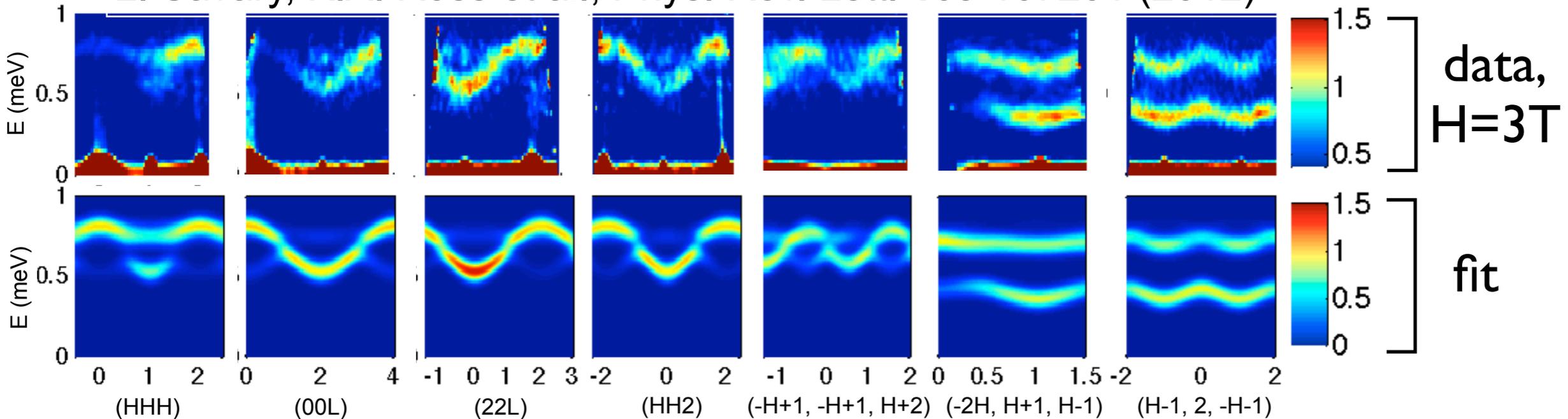
K.A. Ross, et al., Phys. Rev. X **1**, 021002 (2011)



| | Yb ₂ Ti ₂ O ₇ | Er ₂ Ti ₂ O ₇ |
|-----------|--|--|
| J1 | -0.09 | 0.11 |
| J2 | -0.22 | -0.06 |
| J3 | -0.29 | -0.10 |
| J4 | 0.01 | 0.00 |

Er₂Ti₂O₇

L. Savary, K.A. Ross et al., Phys. Rev. Lett. **109** 167201 (2012)



H || to [1-10]

H || to [111]

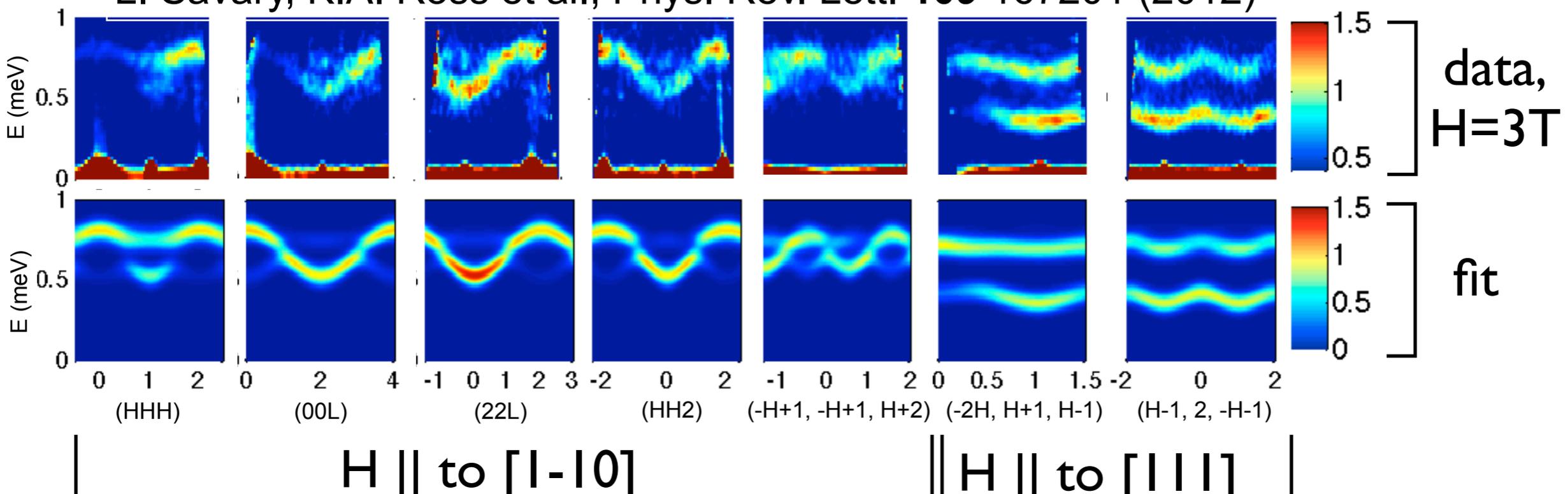
Determining exchange interactions with spin waves from field polarized state

| | Yb ₂ Ti ₂ O ₇ | Er ₂ Ti ₂ O ₇ |
|-----------|--|--|
| J1 | -0.09 | 0.11 |
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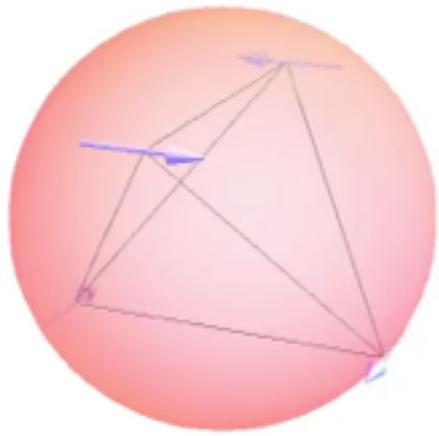
Params lead to "Order by Disorder"

Er₂Ti₂O₇

L. Savary, K.A. Ross et al., Phys. Rev. Lett. **109** 167201 (2012)



Determining exchange interactions with spin waves from field polarized state

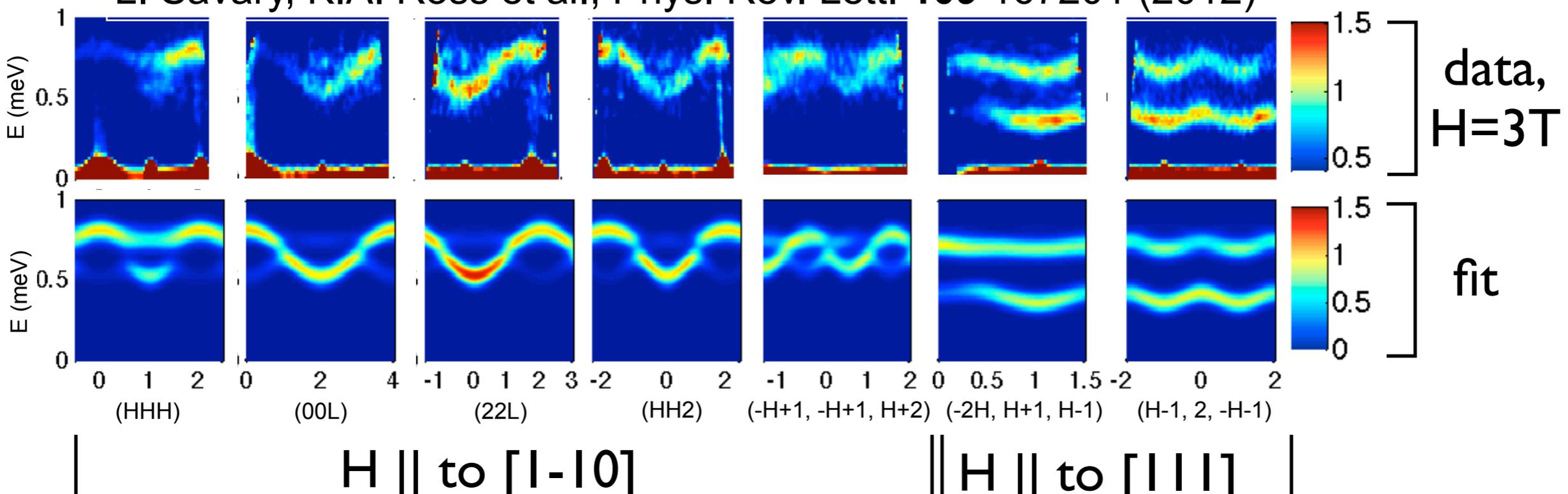


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Er₂Ti₂O₇

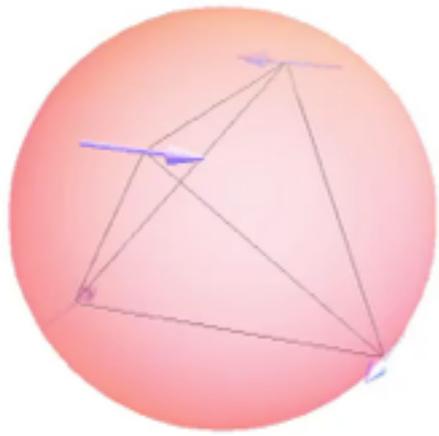
L. Savary, K.A. Ross et al., Phys. Rev. Lett. **109** 167201 (2012)



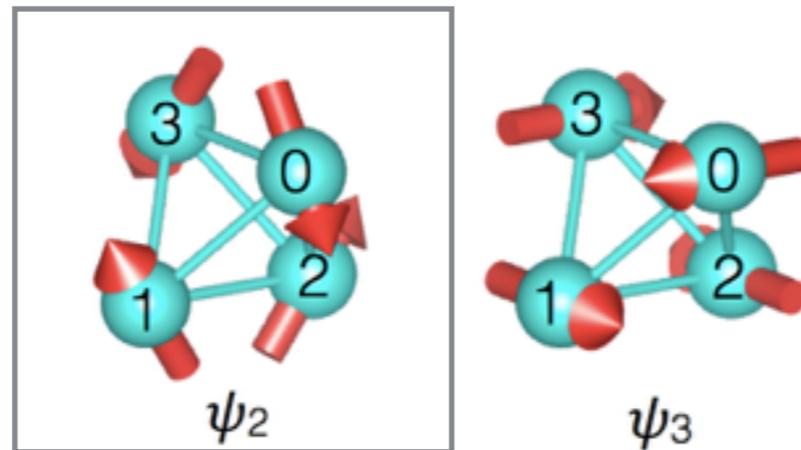
Determining exchange interactions with spin waves from field polarized state

Γ_5 manifold:

Accidentally Degenerate



ψ_2 basis state selected by thermal and quantum fluctuations

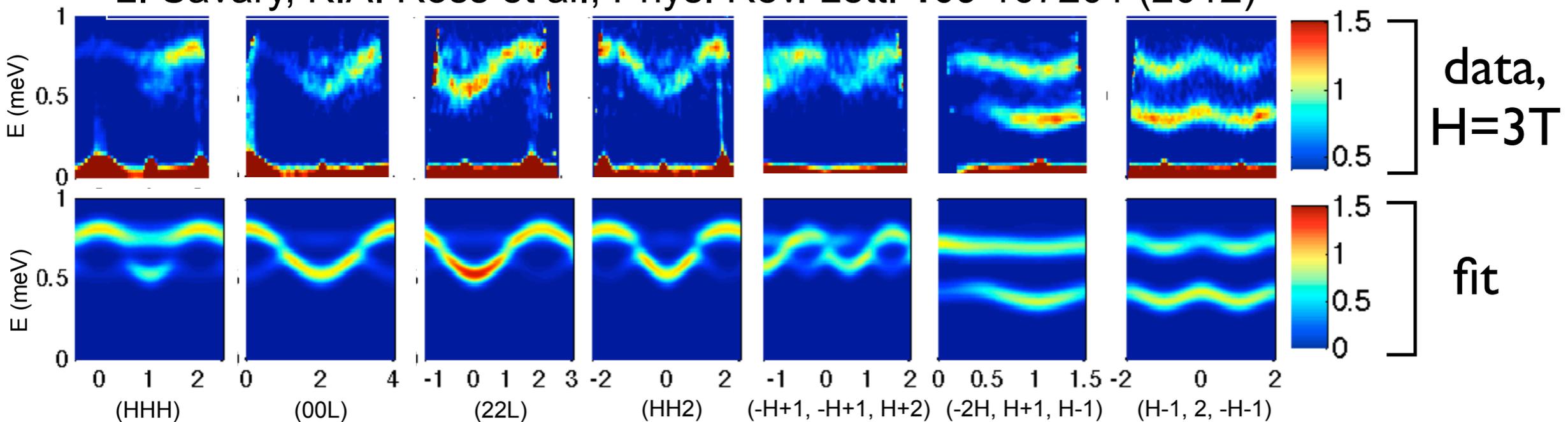


| | Yb ₂ Ti ₂ O ₇ | Er ₂ Ti ₂ O ₇ |
|-----------|--|--|
| J1 | -0.09 | 0.11 |
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L. Savary, K.A. Ross et al., Phys. Rev. Lett. **109** 167201 (2012)

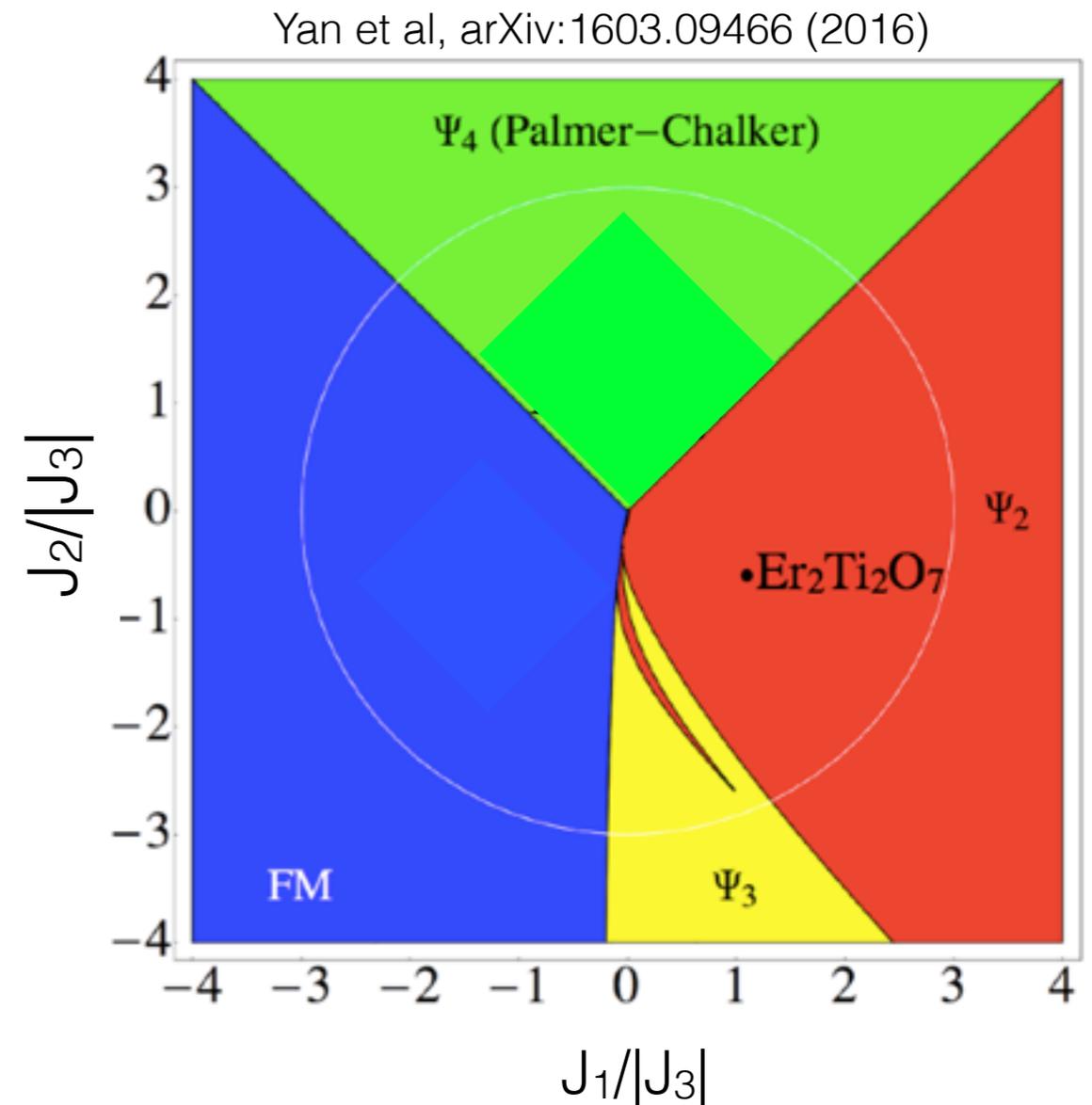


$H \parallel$ to $[1-10]$

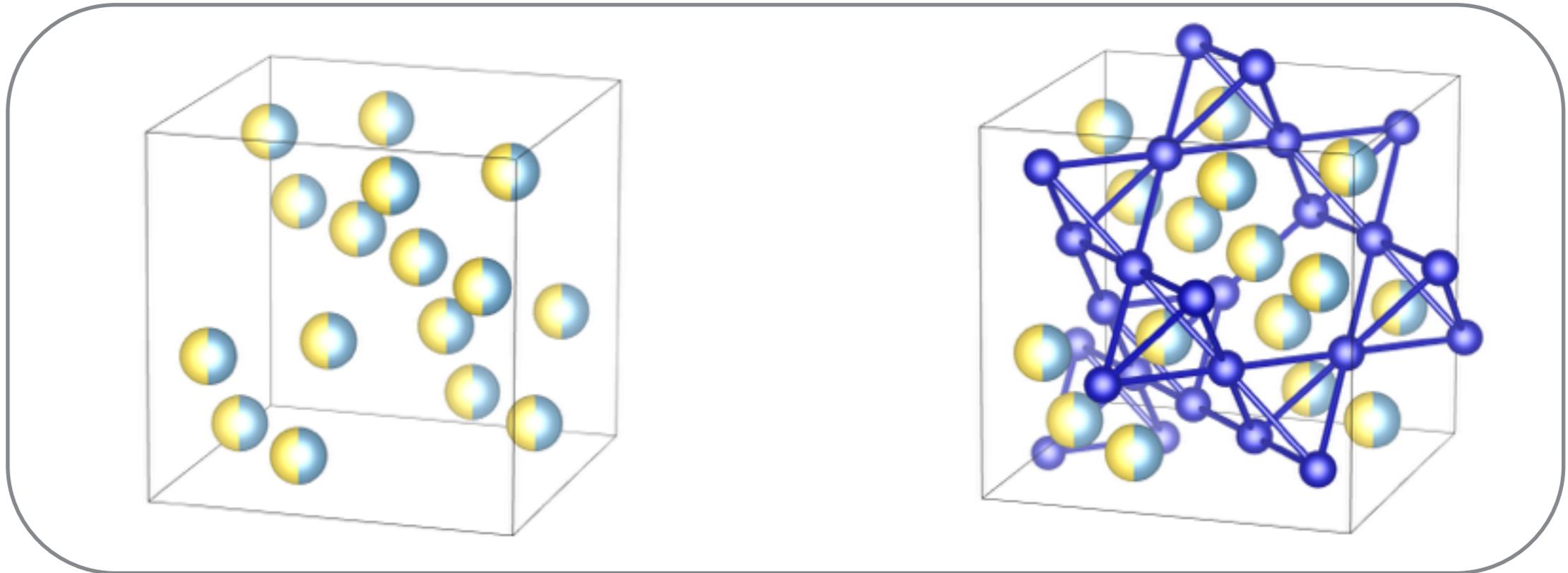
$H \parallel$ to $[111]$

Er₂Ti₂O₇ on phase diagram

- Classical Monte Carlo agrees and puts Er₂Ti₂O₇ squarely inside ψ_2
- **Not as close to the phase boundary**
 - But remember, *thermal* (and quantum) *fluctuations put it there*
- Helps to explain why **no sample dependence** has been observed over the 12 years it's been studied! (despite similar synthesis as Yb₂Ti₂O₇)



NaCaCo₂F₇ and NaSrCo₂F₇

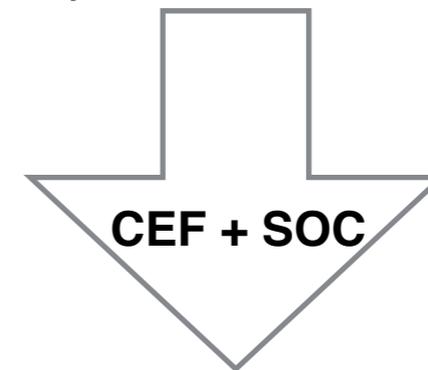


Na⁺/Ca²⁺ or Sr²⁺

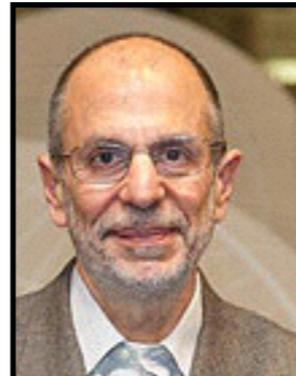
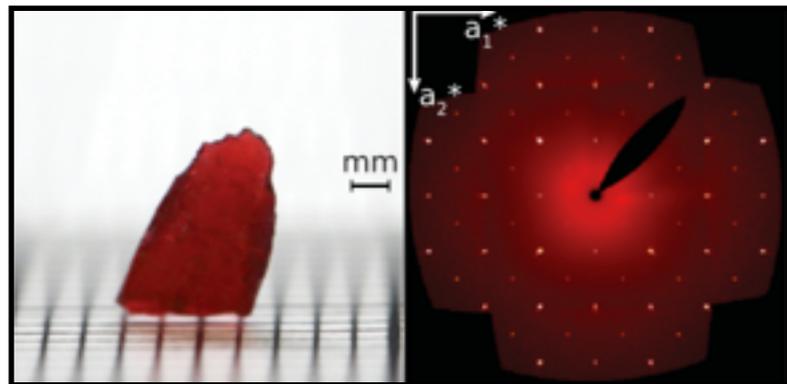
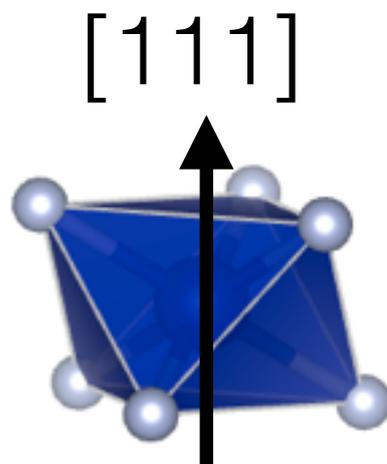
non-magnetic, disordered site

Co²⁺

high spin $S=3/2$, $L=3$



$S_{\text{eff}} = 1/2$

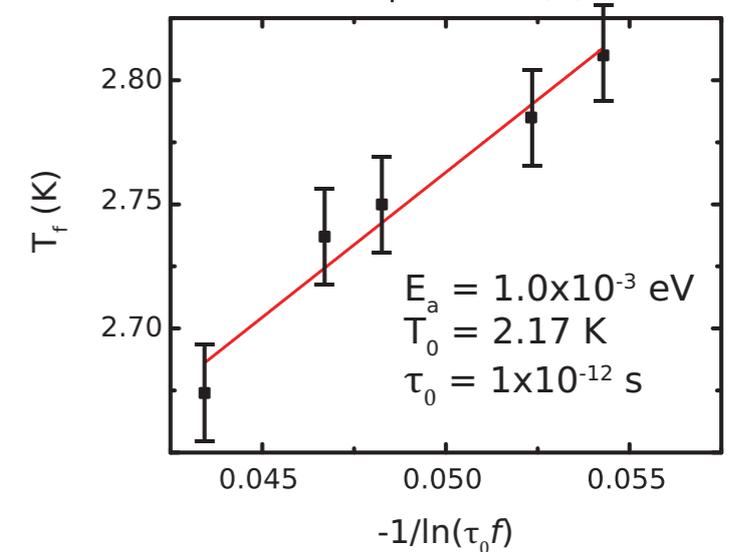
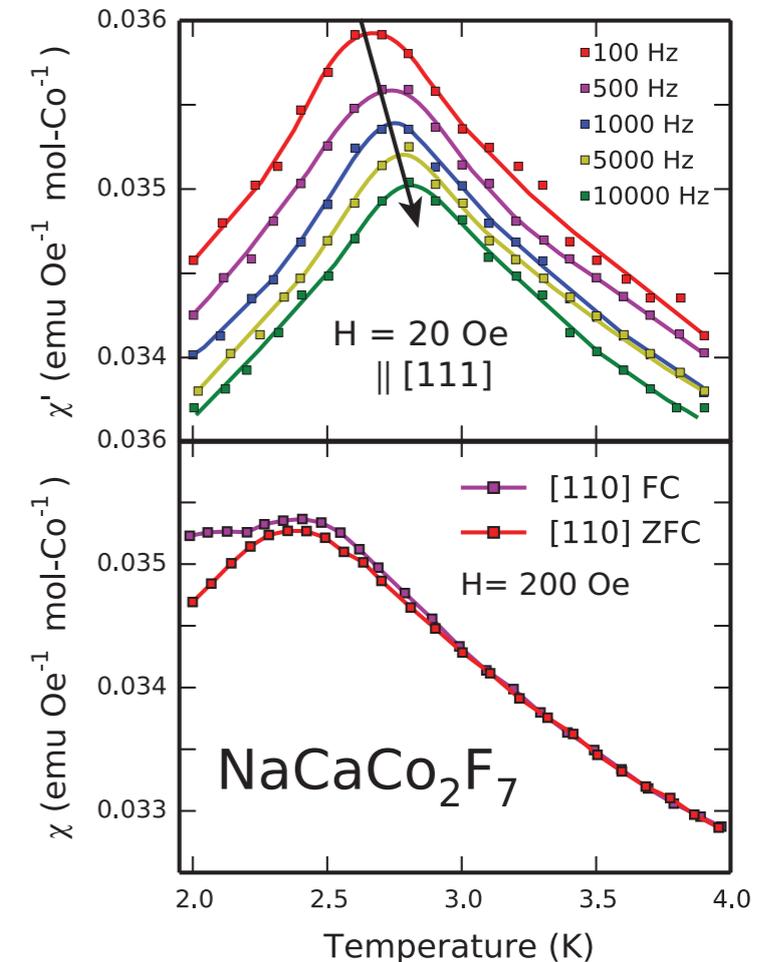
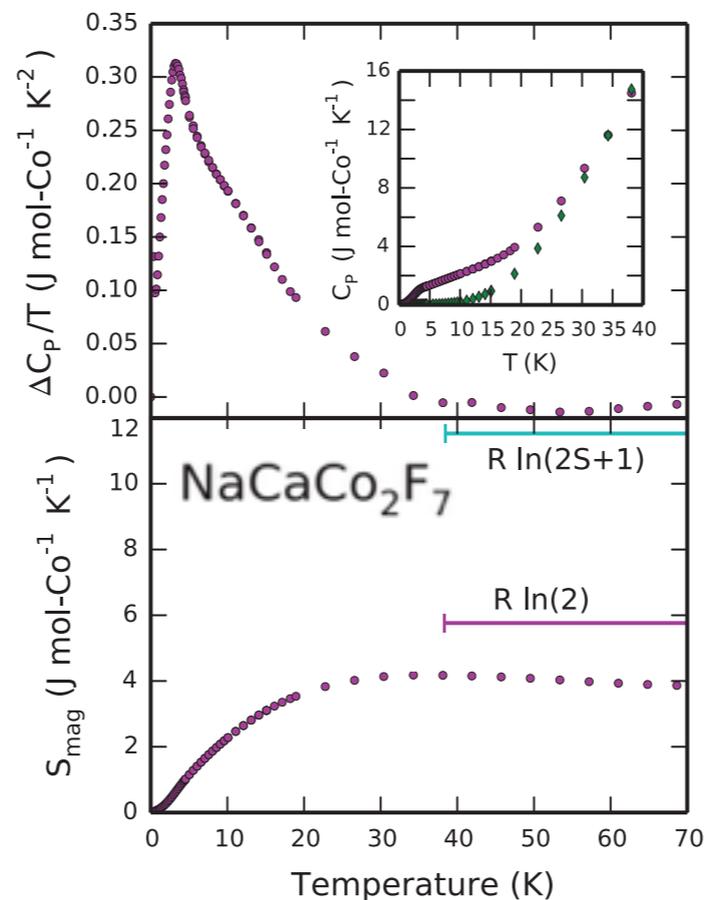
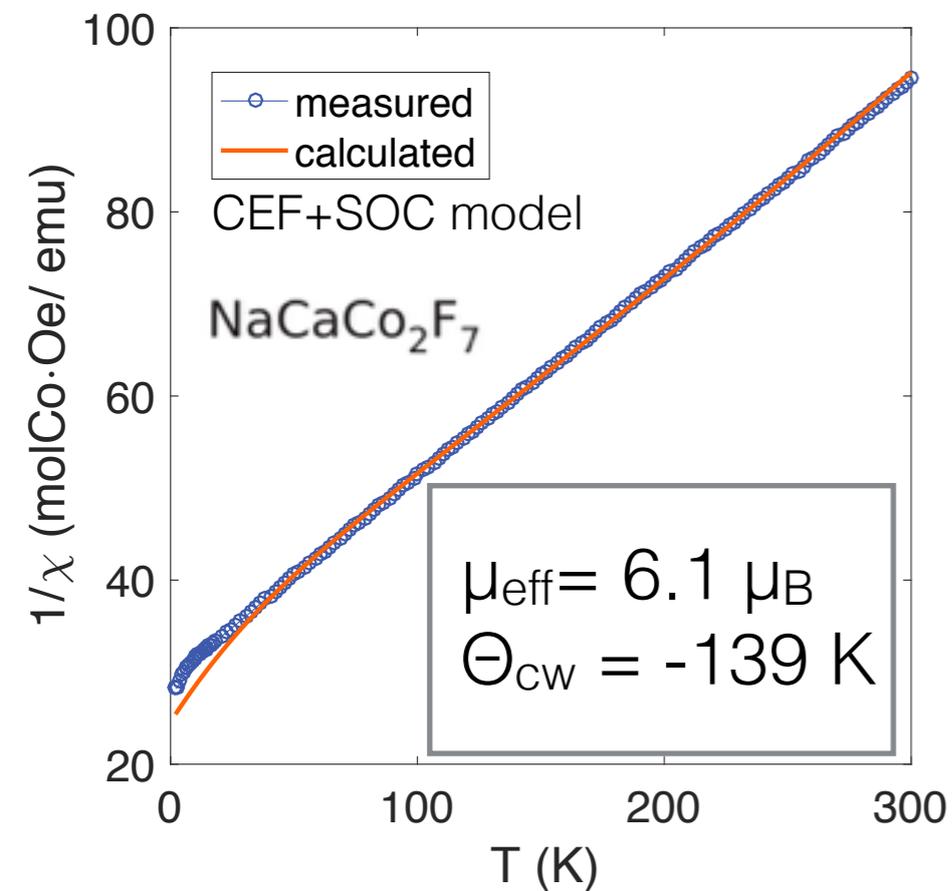


XY AFM pyrochlores with $S_{\text{eff}} = 1/2$

Strong AFM interactions

Broad feature in C_p
Releases $R \ln(2)$ entropy

Spin freezing at
 $T_F = 2.4$ K



Single ion analysis:
K.A. Ross, et al,
PRB 95 (2017)

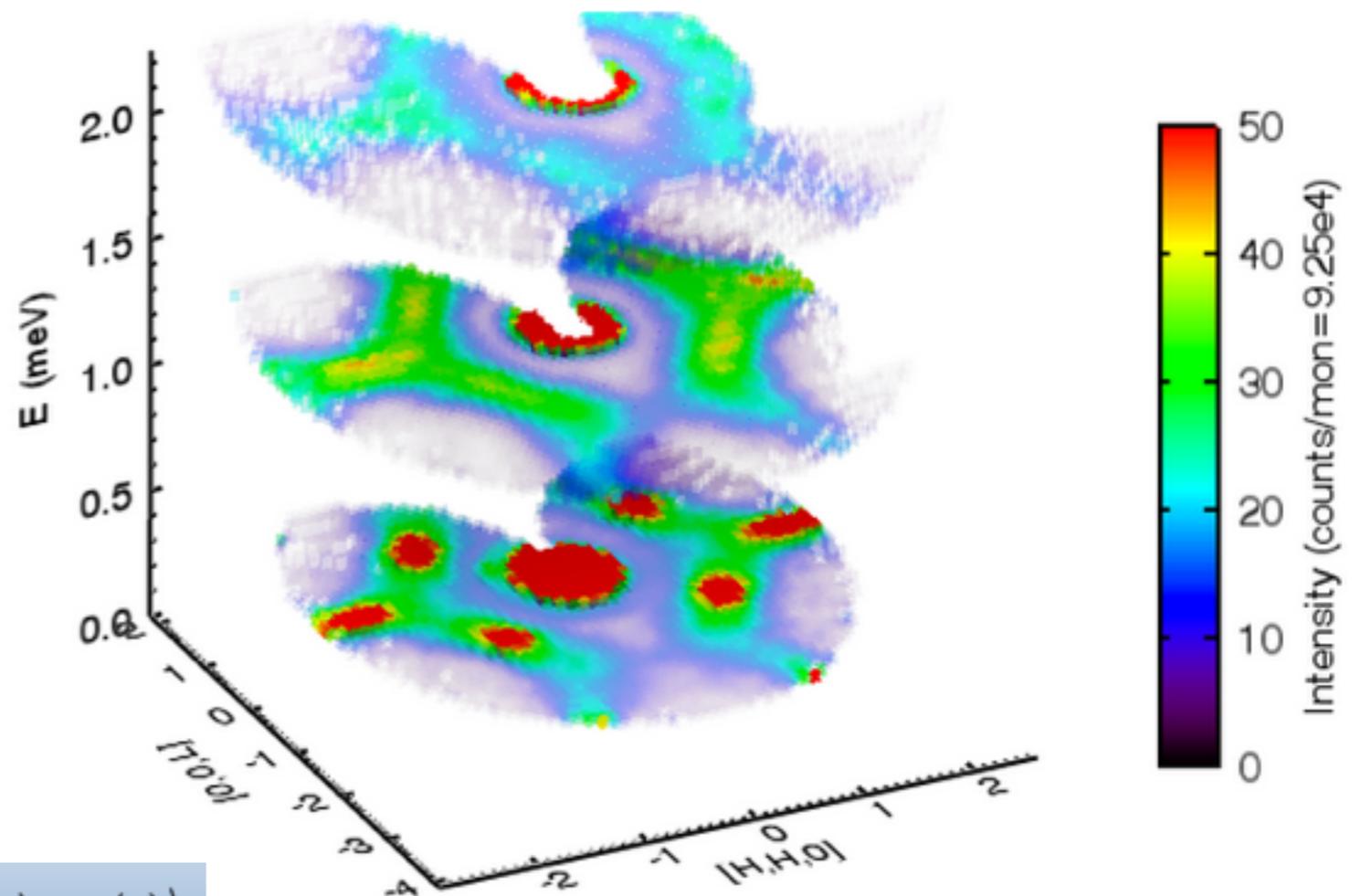
Crystal growth and thermodynamics:
J.W. Krizan, R.J. Cava,
PRB **89**, (2014)

Neutron scattering from frozen state

zig-zag pattern persists to finite energies

It is telling us about the **low energy fluctuations** in the **thermal spin liquid state**

- With **MACS**, we build up $S(\mathbf{Q},\omega)$ using constant energy slices
- Can we interpret the \mathbf{Q} dependence of finite energy slices?
- **Fourier components of spin fluctuations** away from static configuration

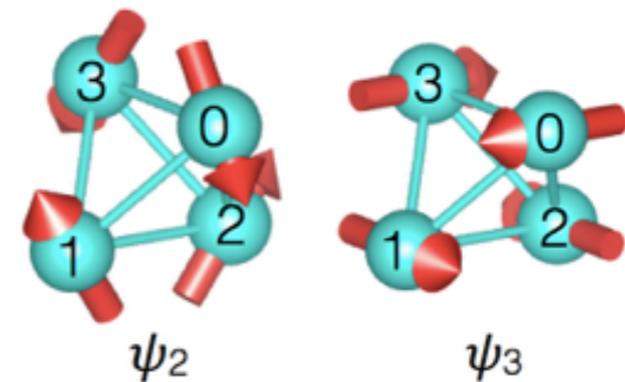
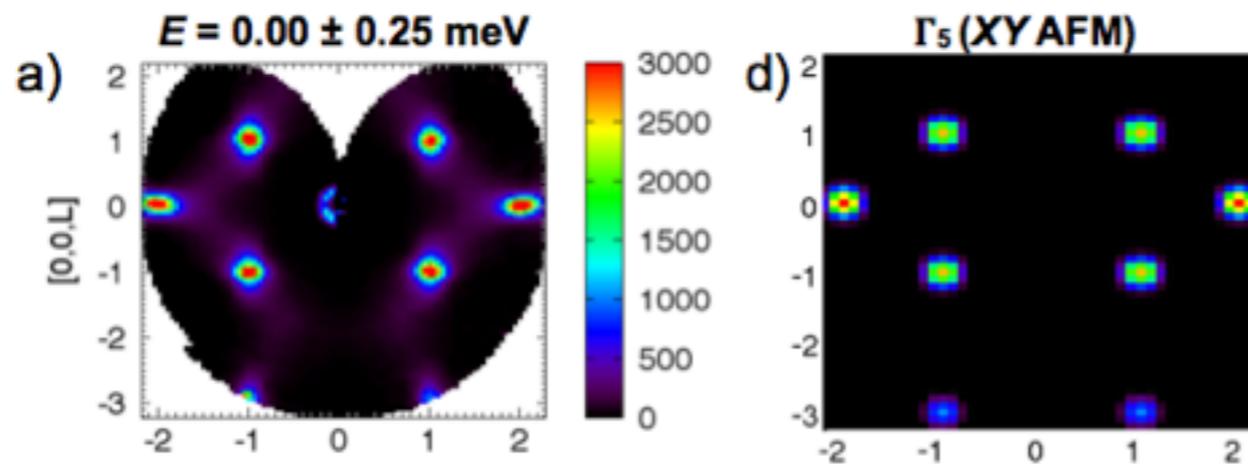


$$S^{\alpha\beta}(\mathbf{Q},\omega) \equiv \frac{1}{2\pi\hbar} \int dt e^{-i\omega t} \frac{1}{N} \sum_{ll'} e^{i\mathbf{Q}\cdot(\mathbf{r}_l - \mathbf{r}_{l'})} \langle S_l^\alpha(0) S_{l'}^\beta(t) \rangle$$

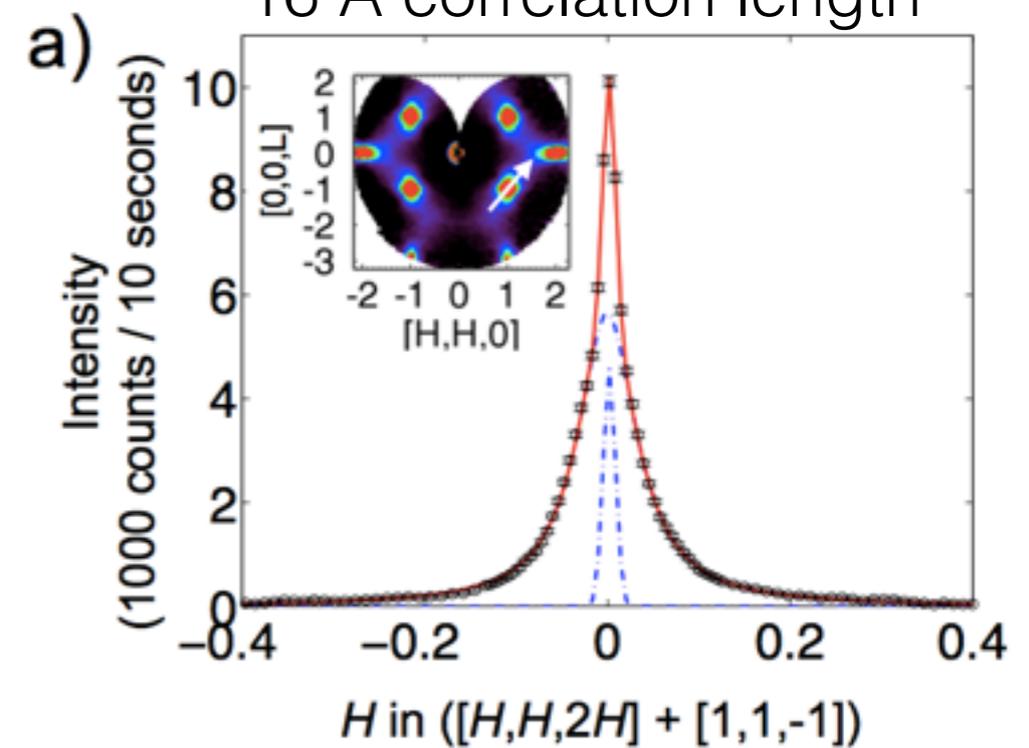
$T = 1.7\text{K}$

Inelastic scattering in frozen state

Same manifold of states as $\text{Er}_2\text{Ti}_2\text{O}_7$!



16 Å correlation length

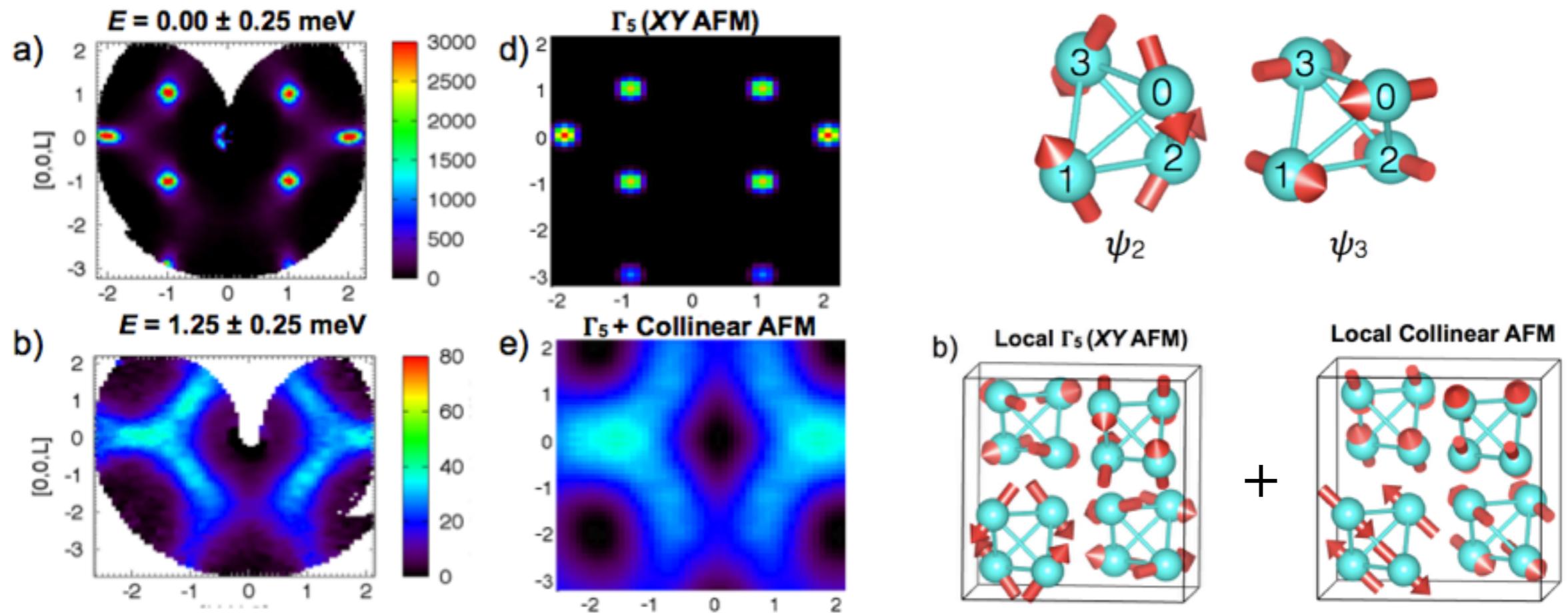


MACS spectrometer NCNR

Ross, et al.. Phys. Rev. B
93, 014433 (2016)

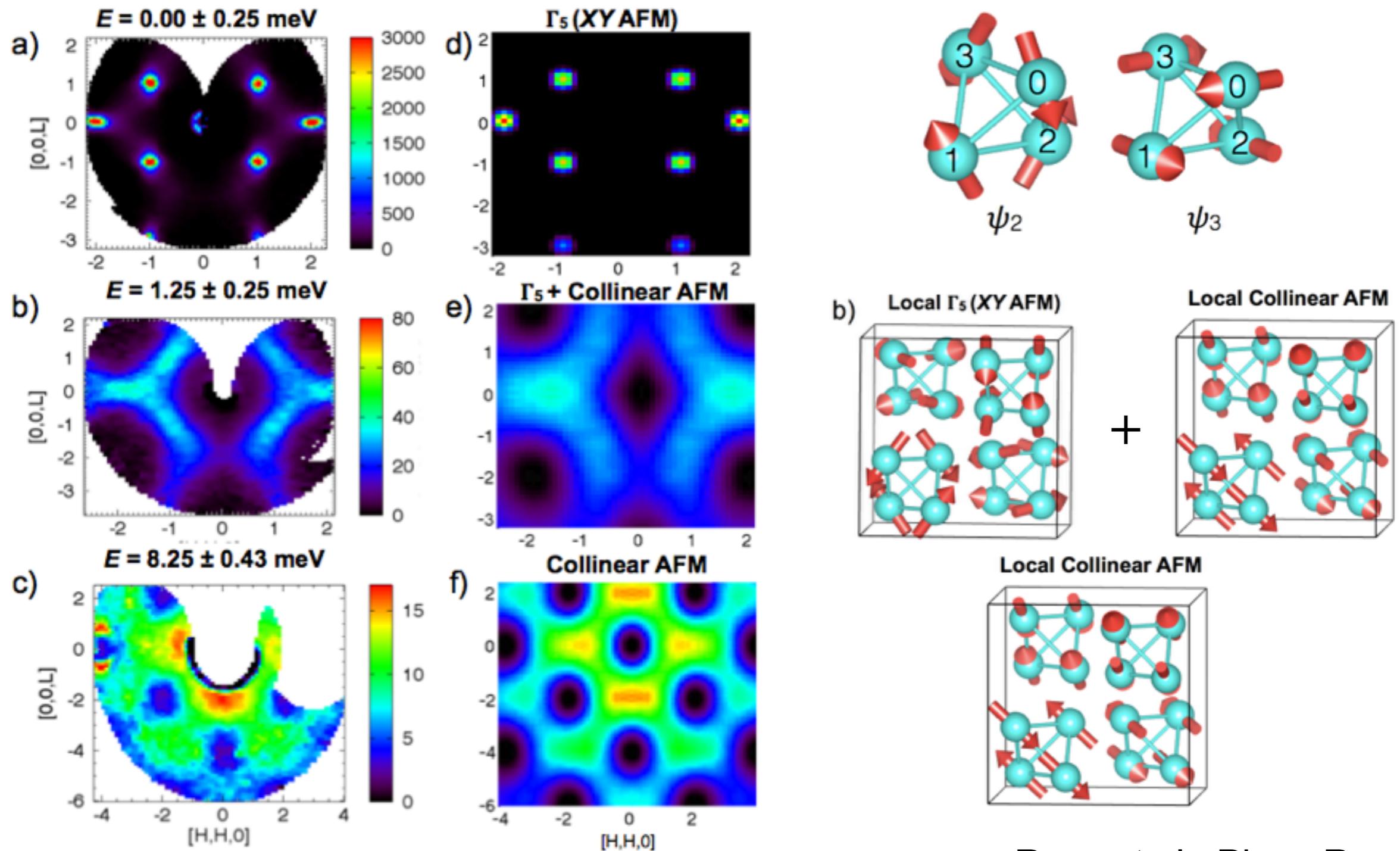
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Inelastic scattering in frozen state

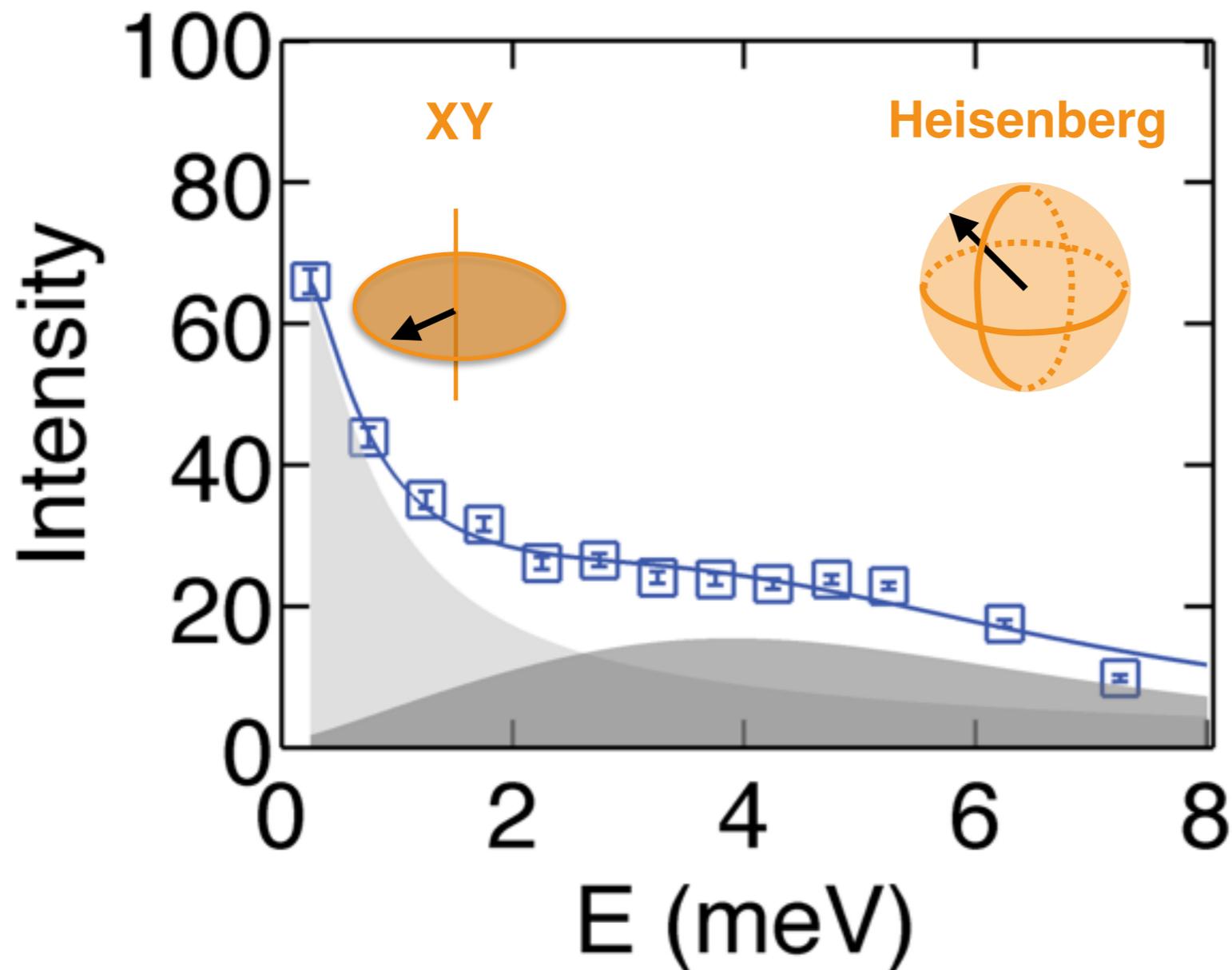
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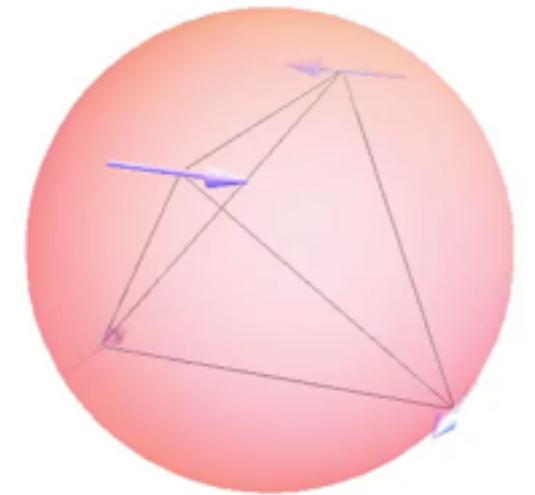
MACS spectrometer NCNR

Ross, et al.. Phys. Rev. B
93, 014433 (2016)

Damped Spin Excitations

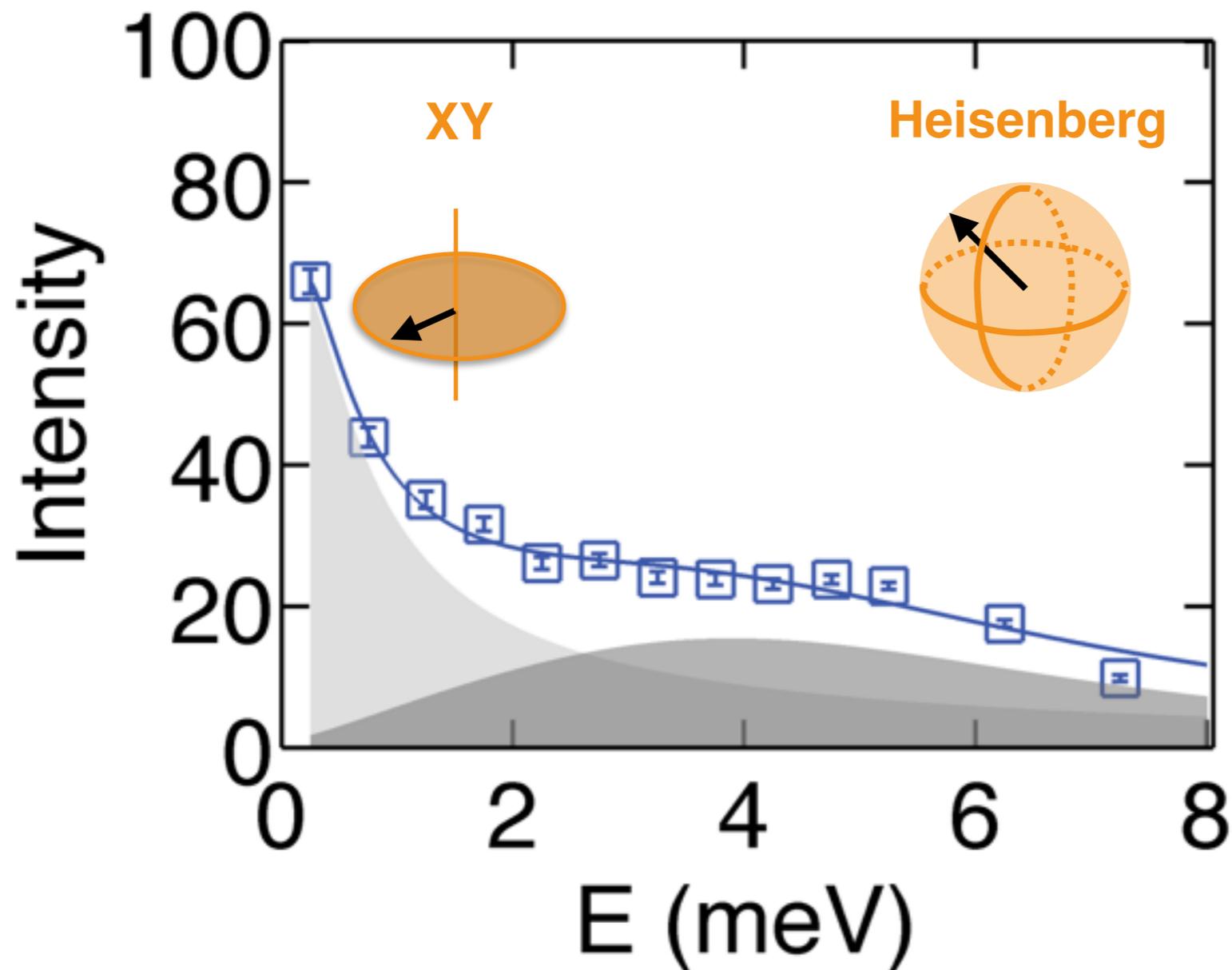


- XY clusters relax slowly: explore the continuous manifold

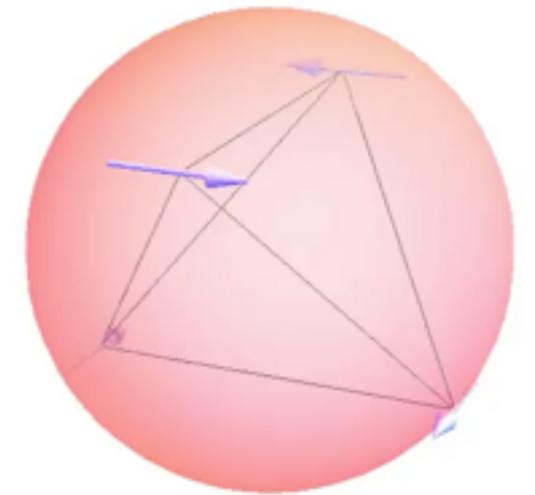


- Crossover to **Heisenberg local mode** at 5.5 meV

Damped Spin Excitations

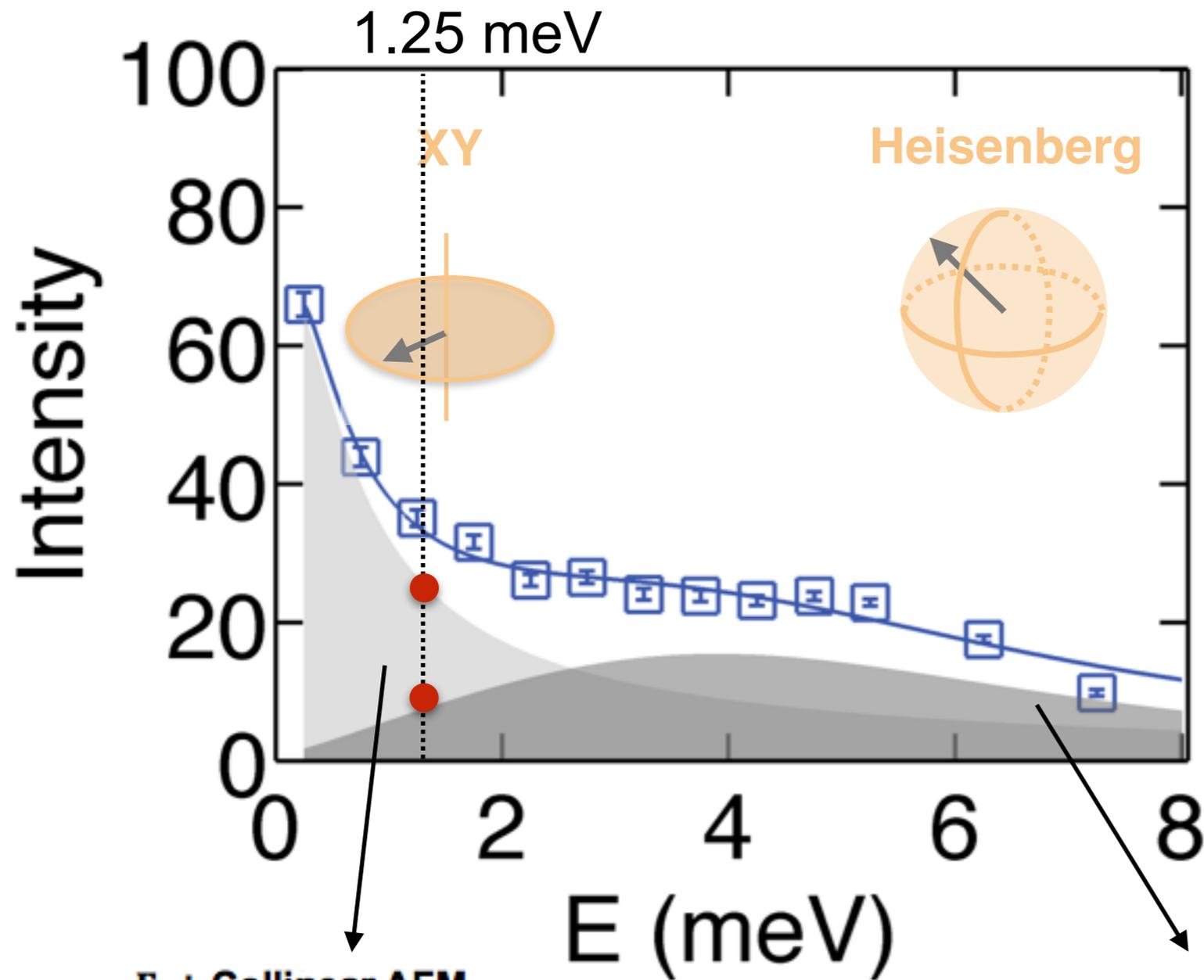


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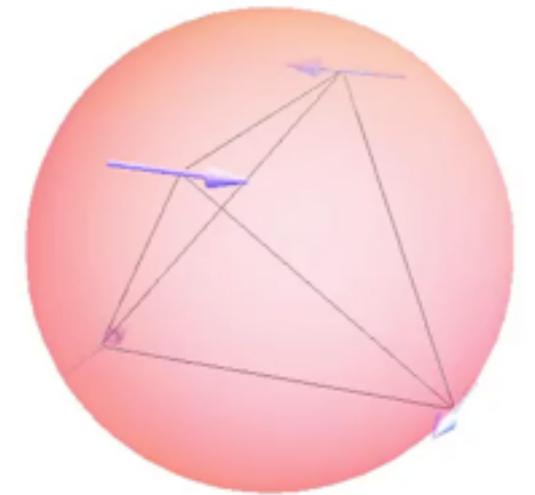


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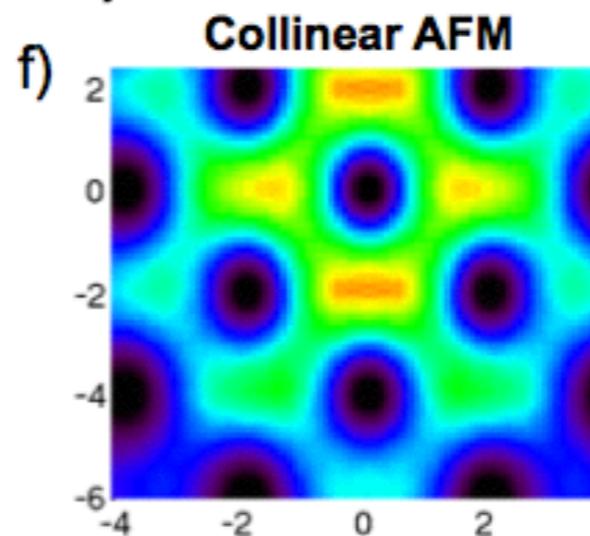
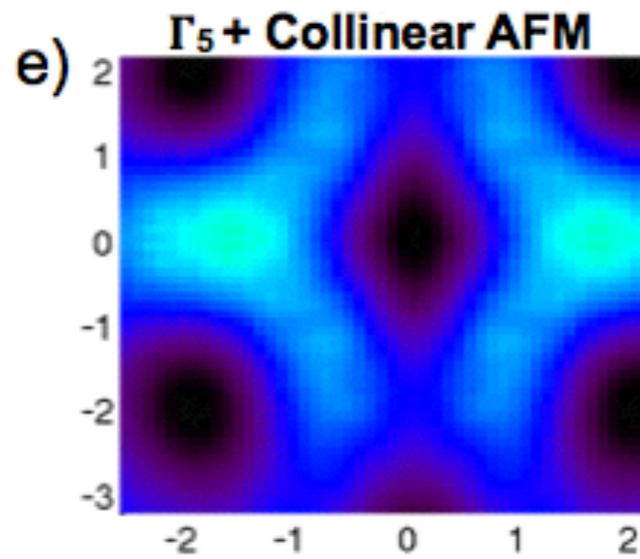
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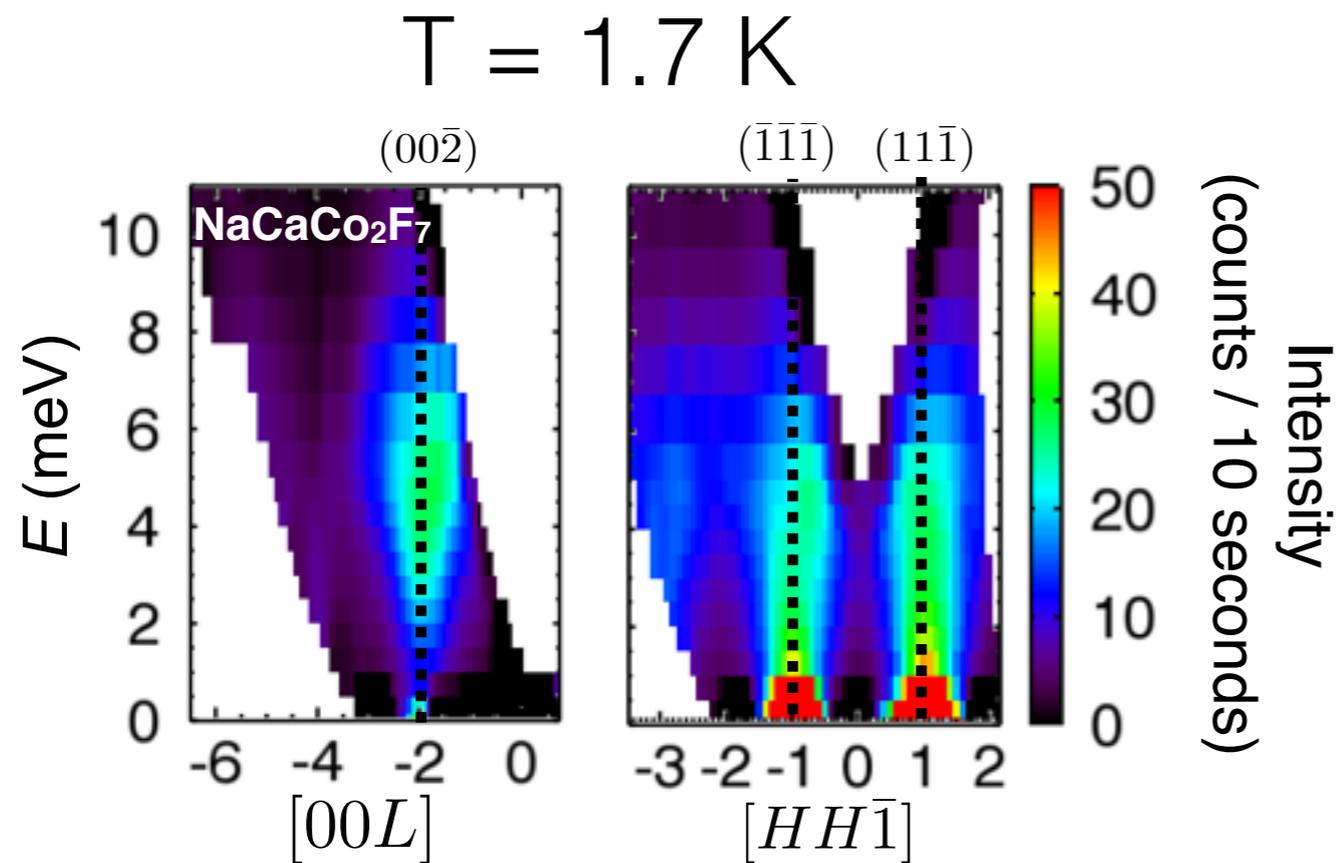
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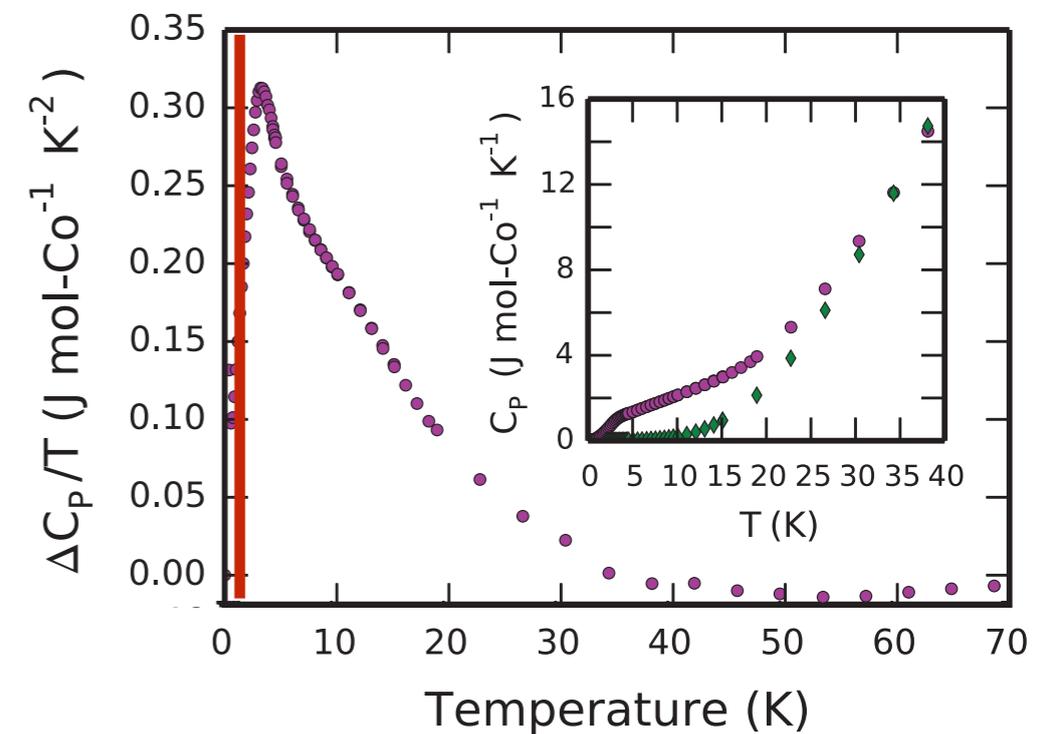
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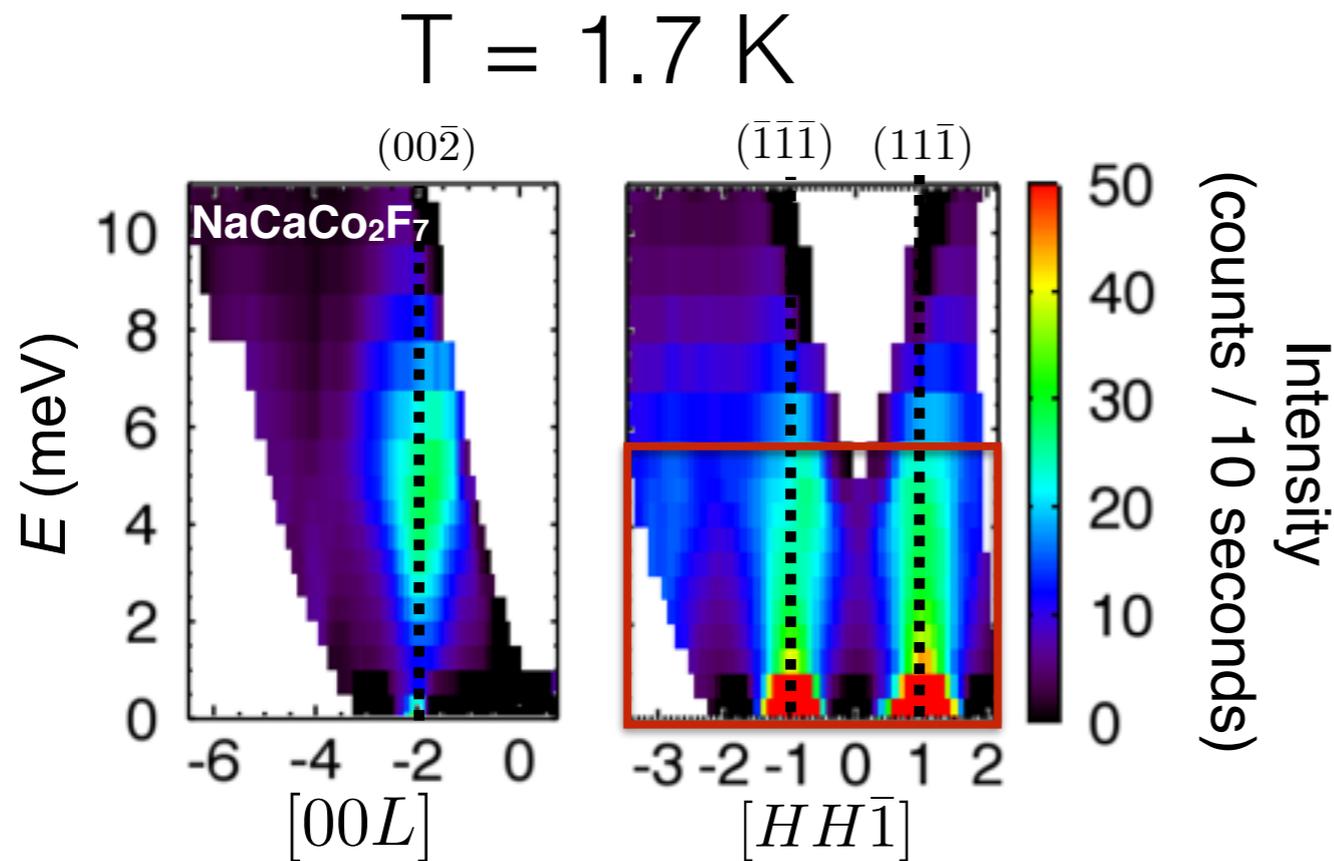
Above T_F , thermal spin liquid



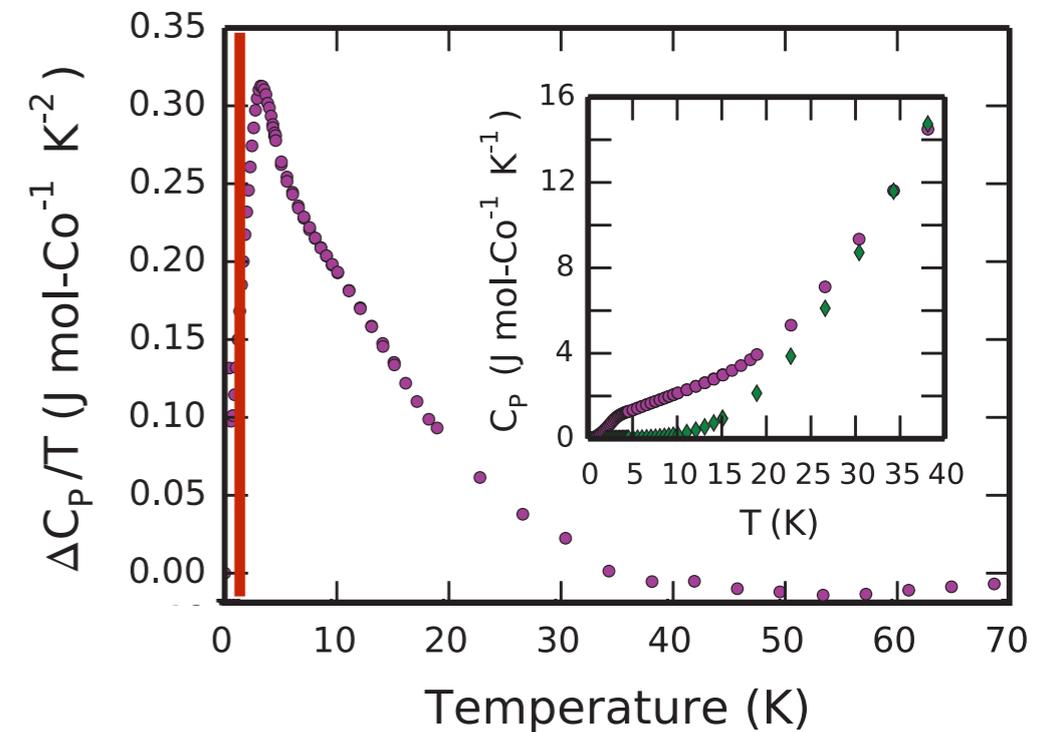
- **Broad excitations** consistent with strongly correlated paramagnet
- Above T_F , Thermal spin liquid over range of nearly 140 K
- **Highly frustrated:**
 - $f = \theta_{CW}/T_F = 58$ in $\text{NaCaCo}_2\text{F}_7$
 - compared to $f = 18$ in $\text{Er}_2\text{Ti}_2\text{O}_7$



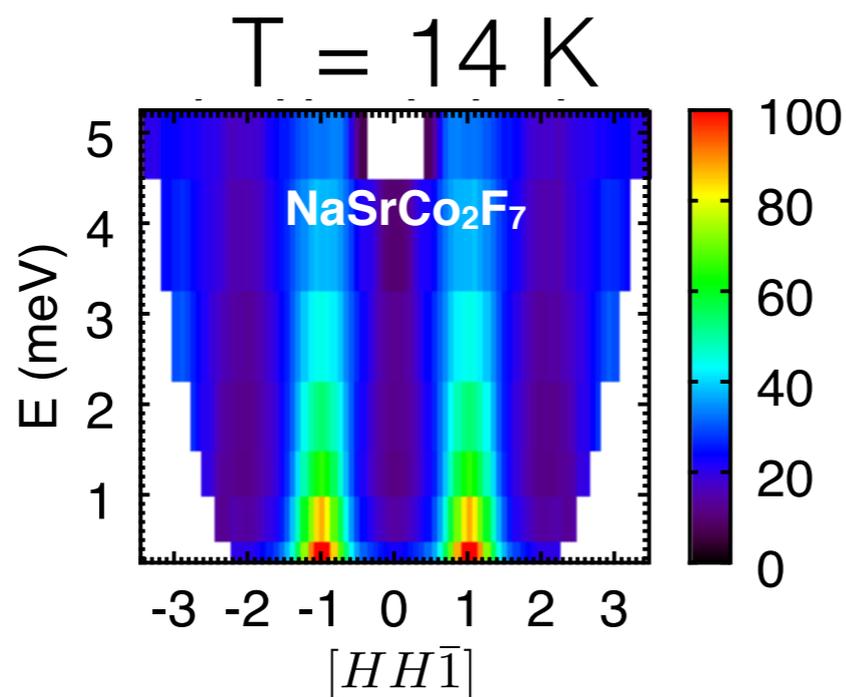
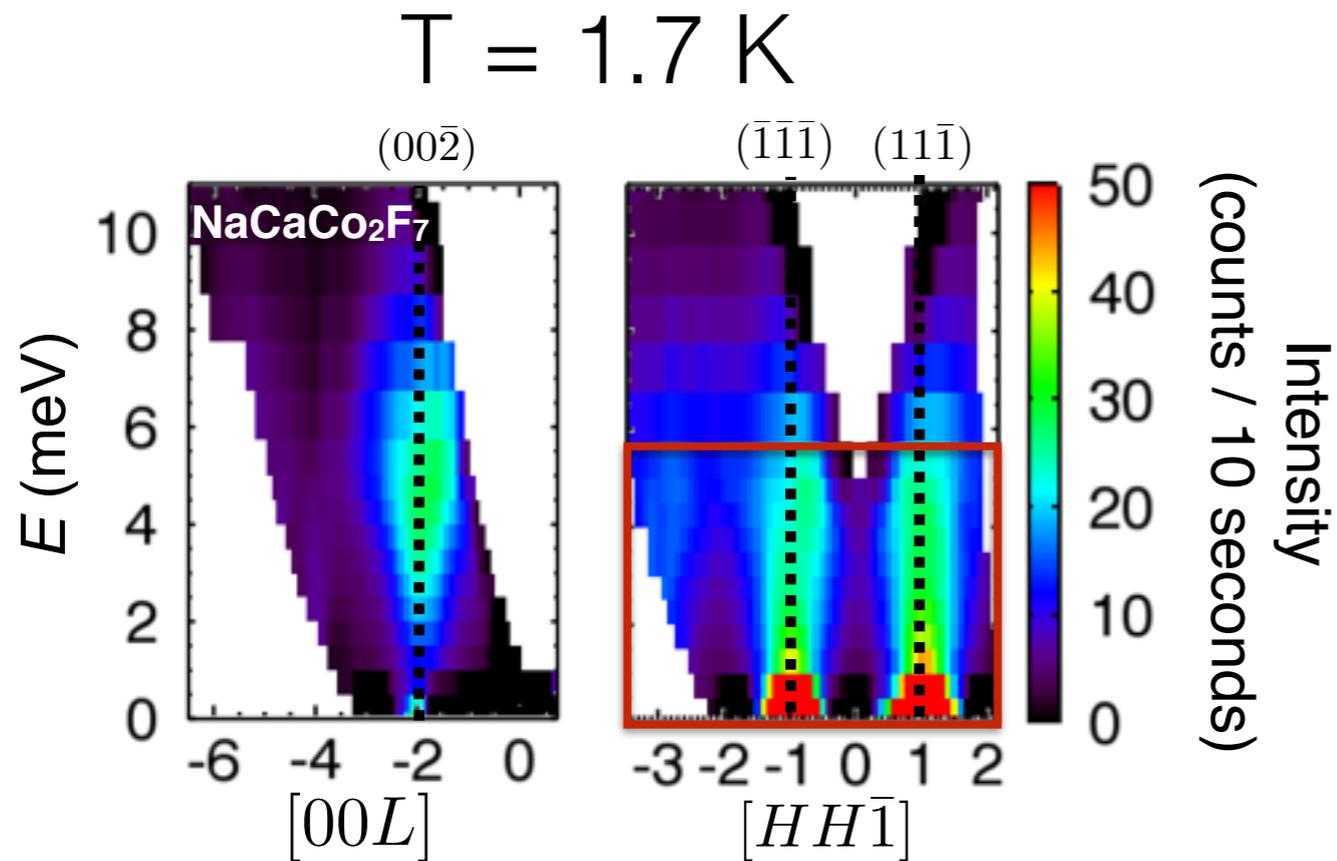
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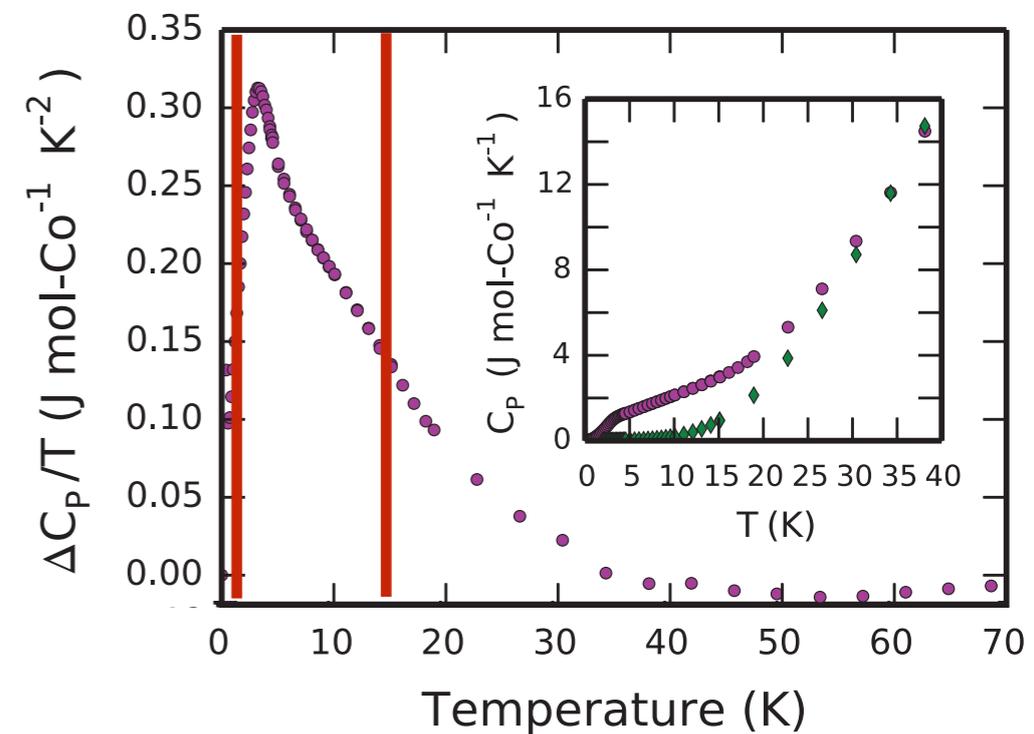
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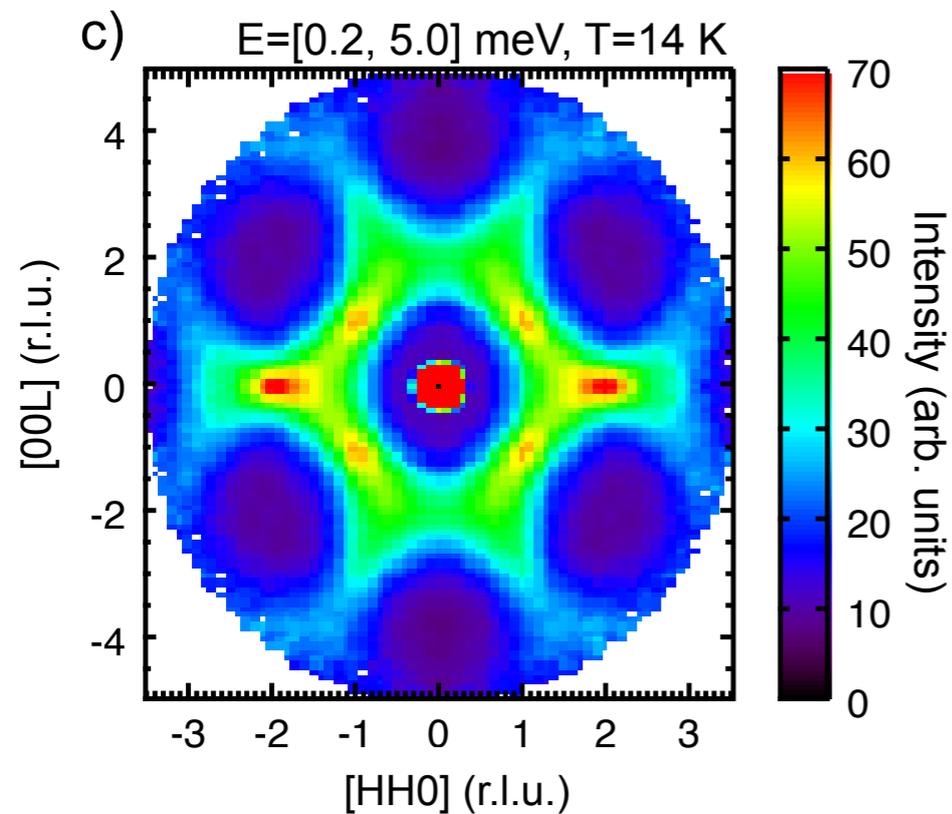
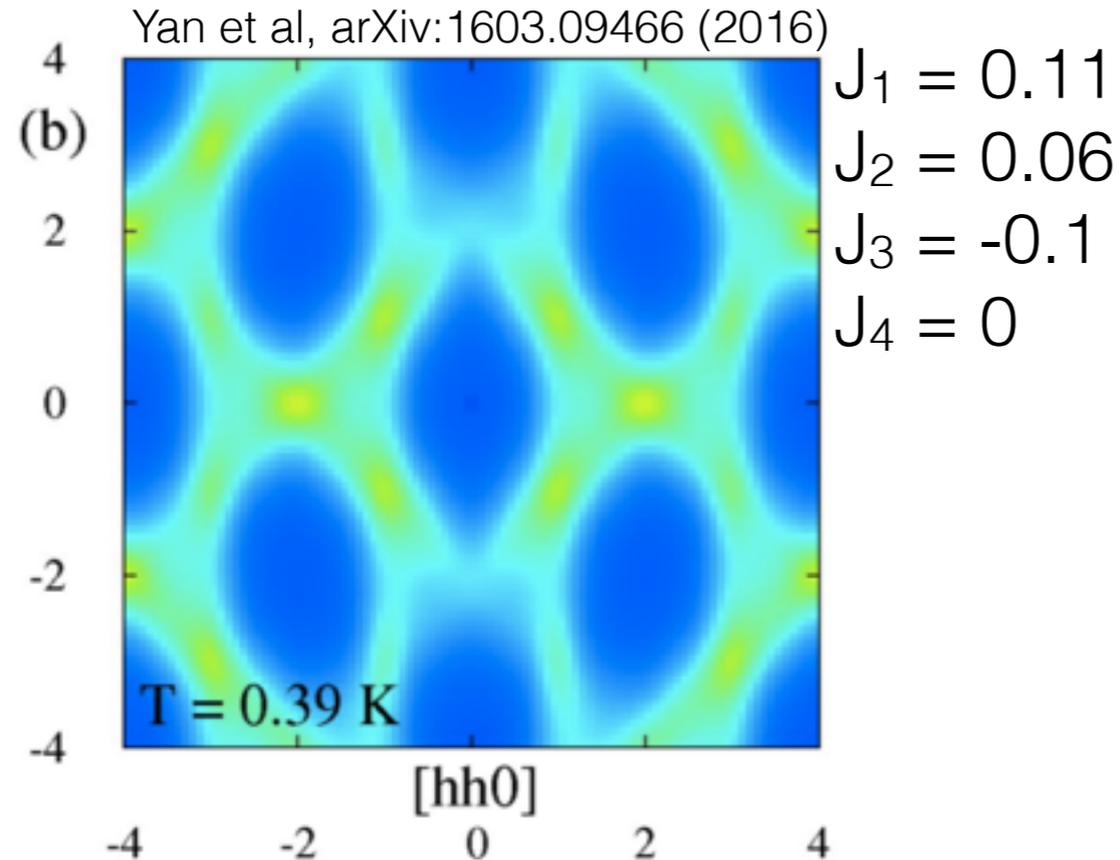
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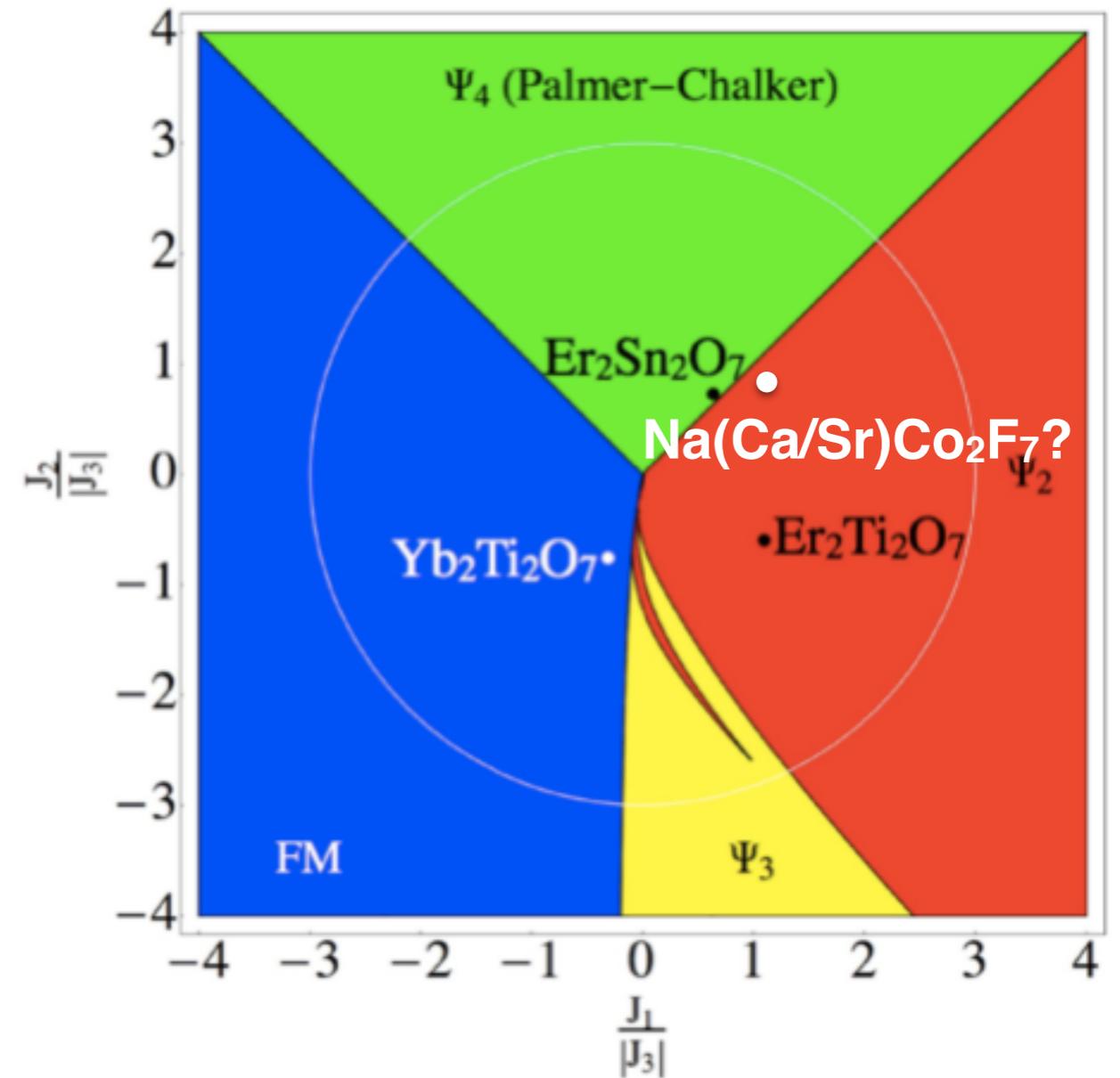
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Comparison to Effective-Spin 1/2 Hamiltonian



$S_{\text{eff}} = 1/2$ with
 XY anisotropic exchange



Yan et al, PRB **95**, 094422 (2017)

Summary

- Frustrated pyrochlore materials based on R^{3+} earths and Co^{2+} act as **effective $S=1/2$** - the “most quantum” they can be
- The same **spin orbit coupling** effects responsible for establishing $S_{\text{eff}}=1/2$ also lead to **anisotropic exchange**
- A general 4-parameter anisotropic exchange model can be used to describe $S_{\text{eff}}=1/2$ pyrochlores
- **Neutron scattering** allows us to probe the diffuse magnetic scattering and field-polarized spin waves, to extract parameters for real pyrochlores, and understand spin correlations
- **Amazing diversity of ground states** can be understood from the deduced relative positions in the **unified phase diagram**
- **MACS, DCS, SPINS** - cold neutron spectrometers used for these studies

Thanks to...

Collaborators

- Bruce Gaulin
- Edwin Kermarrec
- Jonathan Gaudet
- Lucile Savary
- Leon Balents
- Collin Broholm
- Bob Cava
- Jason Krizan
- Steve Nagler
- Matt Stone
- Jan Kycia
- Jeff Quilliam
- Juscelino Leao
- Jose Rodriguez
- Nick Butch
- Leland Harriger
- Yiming Qiu

Funding

- NSERC (McMaster)
- Department of Energy (JHU, Princeton, SNS)
- NSF (CHRNS)
- Colorado State University (startup)